Contents

1 Topological Defects

1.1 Definition

A topological defect is characterized by a core region (i.e. point/line) where order is destroyed and a far field region where the elastic variable changes slowly.

Remark.

- It's presence can be determined by non-local measurements.
- Plays an important role in determining macroscopic properties of the material (e.g. mechanical properties of steel, phase transitions in 2 dimensions from low-temperature non-vanishing rigidity phase to a high temp disordered phase).
- Some top defects are vortex (characterized by winding), point defects (hedgehog defects in 3d), disclinations (nematic liquid crystals) and dislocations (periodic crystals).
- Are topologically stable : can't be made to disappear by any continuous deformation (meaning a sequence of deformations that slightly change the OP at each point wrt to previous configuration).
- Are *generally* physically stable : can't be taken to GS without encountering a high-energy configuration in between. Physical stability and topological stability are **distinct**.

1.2 Skyrmions

Example problem

- Topological solitons: topological configurations (with quantized charges) that are stable to smooth deformations of the OP configuration.
- Don't involve a singularity in the OP field and has intensive (wrt system size) energy scaling. Wavefunction strongly localized near the centre of the soliton.
- Skyrmion: 2 or 3 dimensional topological soliton. (in 1d we just call it soliton).
- Example: XY ferromagnet in 1d: Compactifying 1d line, we see that it's a vortex on the

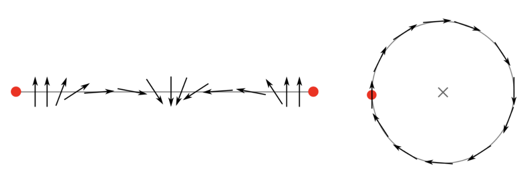

Figure 1: Taken from yk where

compactified space S^1.

- Higher dimensional analogs characterized by $\pi_D(G/H)$. For a 2d plane, it can be compactified into S^2 using sterogrpahic projection and then a S^2 valued OP in monopole/hedgehog configuration can be defined. Unfolding back would give us a skyrmion configuration.

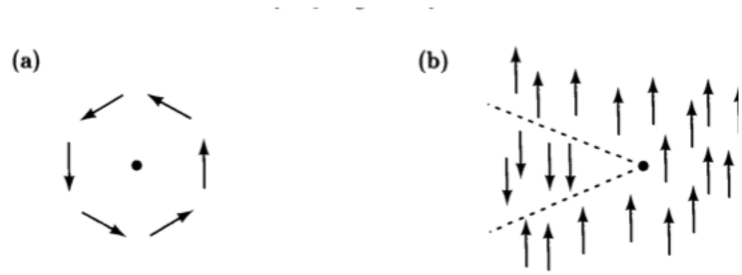

Fig. 9.1.6. (a) A $k = 1$ vortex. (b) An attempt to align spins by a continuous distortion. Note the appearance of boundaries (shown as dashed lines) wher the spin changes rapidly from up to down.

Figure 2: Taken from Chaikin Lubensky Chapter 9
$E_{vortex\ on\ LHS} = JlnR$ while $E_{RHS} = J * L$, L being the sample length.

– Seen in qHall magnets and magnetic materials. Also called baby skyrmions in nuclear physics models.

2 Vortex States: Superfluid He vs s-wave SC

For Superfluid He

1. Circulation quantization: $\oint_C \vec{\mathbf{v}} \cdot d\vec{\mathbf{l}} = \dfrac{\hbar}{m} \cdot 2\pi n \implies \quad v_s \propto \dfrac{1}{r}$ for large r
2. $E_{vortex} = \dfrac{1}{2}\rho_s(\dfrac{nh}{m})^2\ log(R/r_{vortex})$, where R is system size i.e. extensive. Comes from the fact that v_s is power-law (exponential in case of a SC vortex). Force between vortices is power law.
3. Why did we posit it's existence: a rotating HeII bucket was found to have both the viscous normal component (ρ_n and the **inviscid** superfluid component (rho_s rotate. This would mean that $v_s = \vec{\omega} \times \vec{\mathbf{r}} \implies \boldsymbol{\nabla} \times \vec{\mathbf{v}} = 2\vec{\omega} \neq 0$ i.e. violating superfluid condition.
Resolution was that many small vortices form (with non-zero vorticity, as they aren't simply connected) that can give rise to a net vorticity.

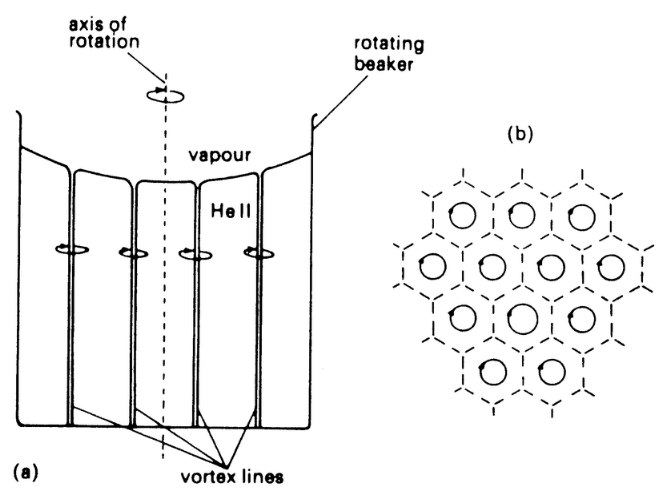

Figure 3: Small vortices produce a net vorticity

4. use the above for onset of vortex criterion: At low rotational angularmomentum of the bucket vortices don't appear. Above a critical $\vec{\mathbf{\Omega}}_c$, vortices can form to reduce the total energy. $\vec{\mathbf{\Omega}}_{\mathbf{c}} = \frac{\hbar}{mR^2} log(R/r_{vortex})$, R being the bucket radius.

5. Above a certain $\vec{\mathbf{\Omega}}_{c2}$, numerous vortices overlap and superfluid component gets destroyed. Rotation in HeII is like as applying a field in type-II SC.

~~What's vortex interaction energy for HeII? See here: q2-4 of Nigel's pset on topological defects~~

For superconductor:

1. Fluxoid quantization: $\oint_C \vec{\mathbf{B}} \cdot d\vec{\mathbf{S}} + \oint_{\partial C} \vec{\mathbf{v}} \cdot d\vec{\mathbf{l}} = \frac{\hbar}{m} \cdot 2\pi n$

2. $E_{vortex} = \frac{\Phi}{2\pi\lambda^2} ln(\frac{\lambda}{\xi})$ i.e. not extensive (but ofc this is energy per unit line). For between vortices is exponential.

3. Why did we posit it's existence: For type II SC, the S/N domain wall energy is negative. Then why doesn't the system break into smaller S/N chunks and we have phase transitions at very small lengthscales (imagine tubules of N, which don't really lose out on f_{cond} but have large surface area to compensate the loss). This would invalidate GL theory too, as it is not applicable at small scales. To save this, we posit existence of vortices which respect fluxoid quantization, that restricts them from being tiny.

4. Similar ideas follow for H_{c2}

See my superconductor vortex states notes for clarification