Non-Centrosymmetric Superconductors

Guru Kalyan Jayasingh
4th year physics undergraduate

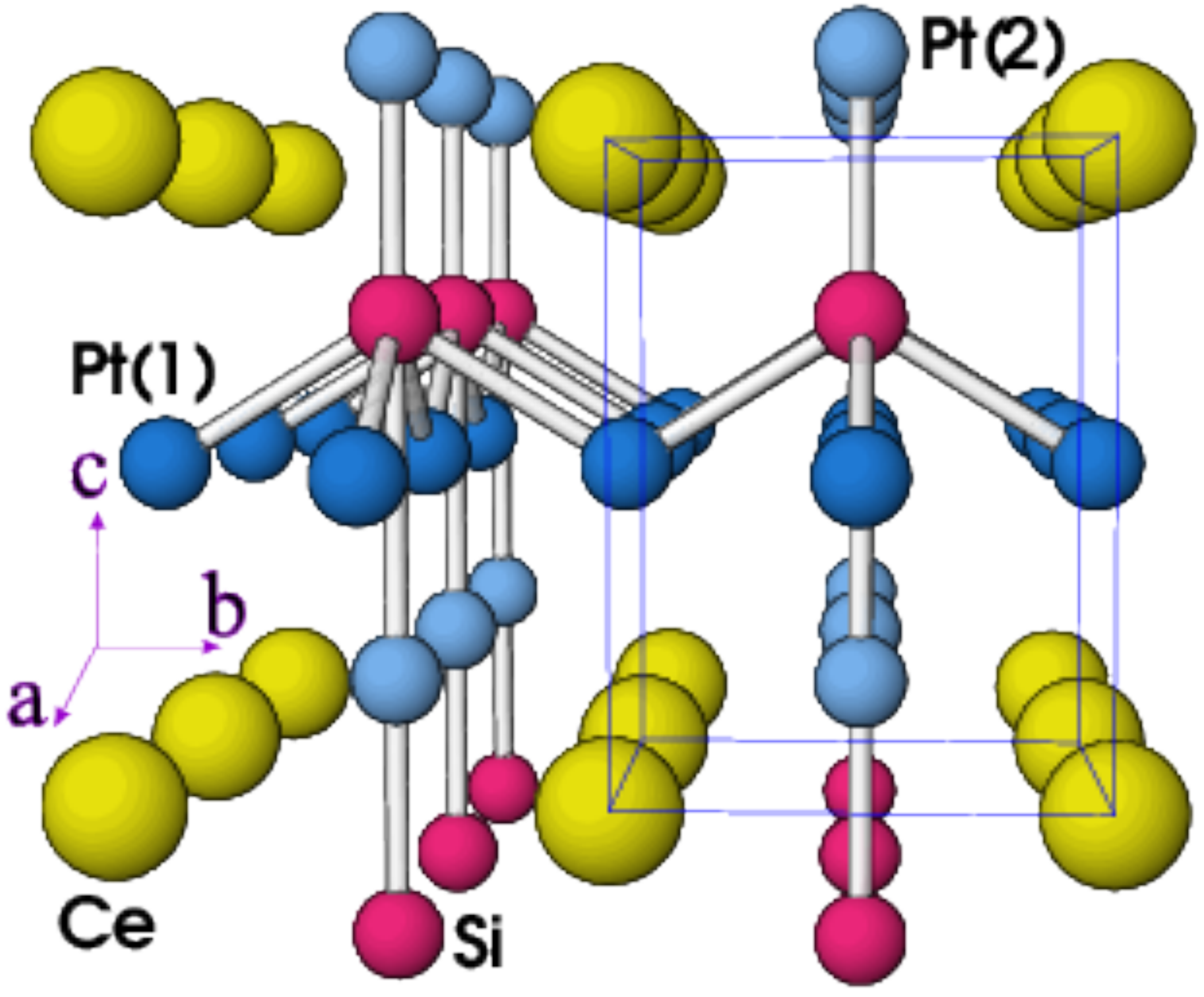
Guided by: Dr. Alexander Zyuzin (DAP, Aalto University, Finland)
Prof. Sumiran Pujari (Dept. of Physics, IIT Bombay)

Presentation for PH 557 BTP stage - I

Plan for today

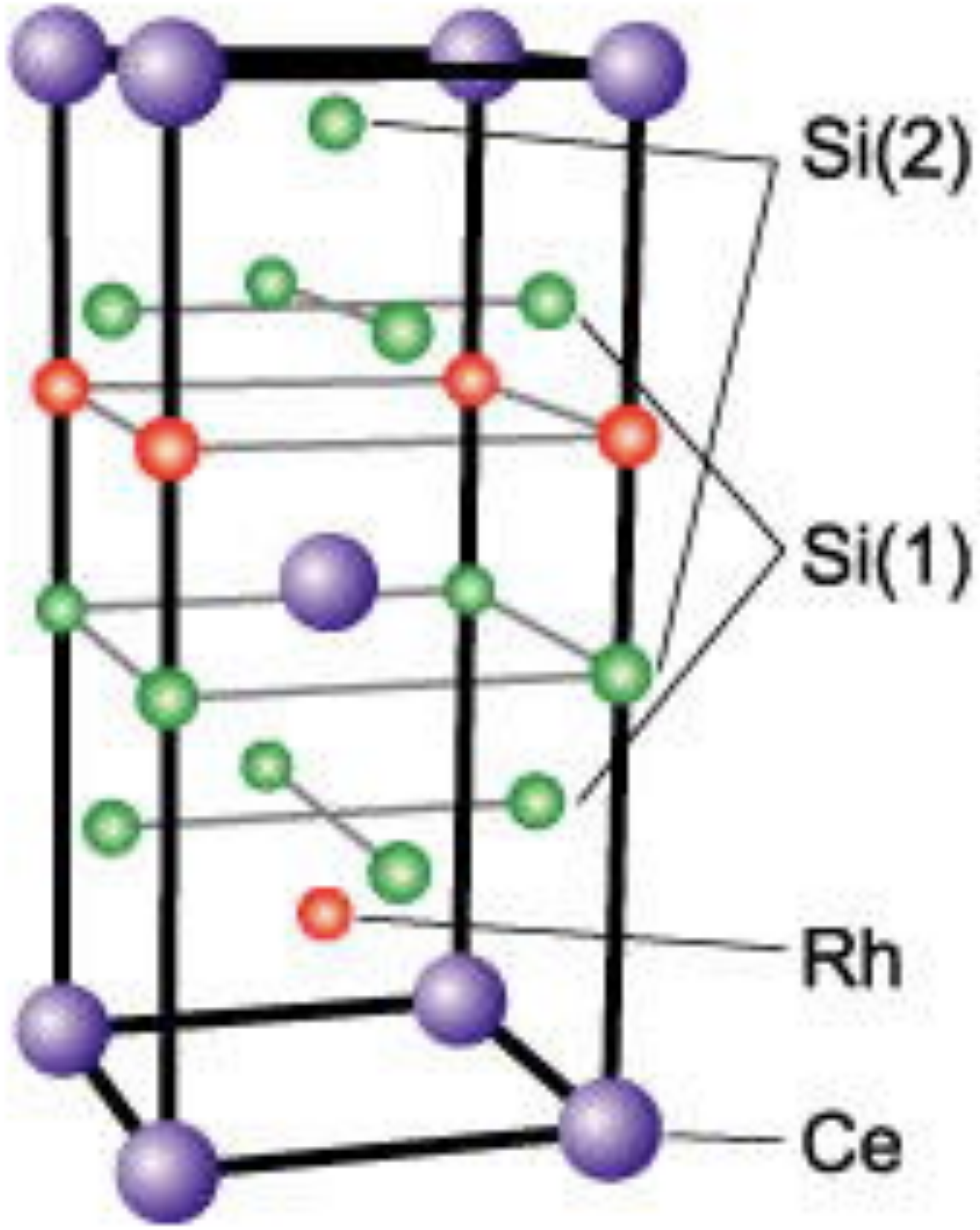
- Spin-orbit coupling
- Paper review: Non-centrosymmetric SC
- Fluctuations in superconductor
- Diamagnetic Susceptibility calculation

Lack of inversion centre



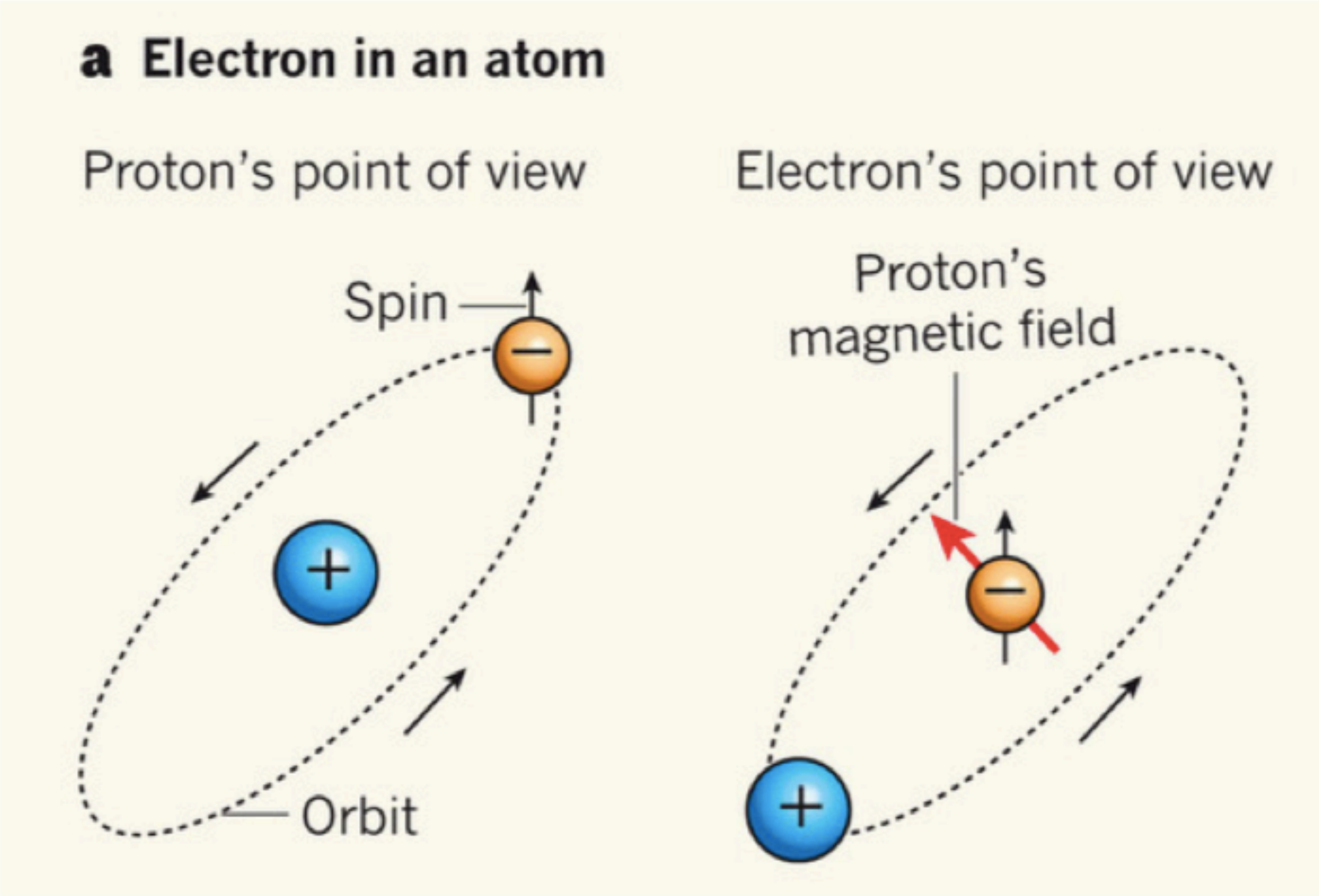
$CePt_3Si$ - P4mm; $CePt_3B$-type

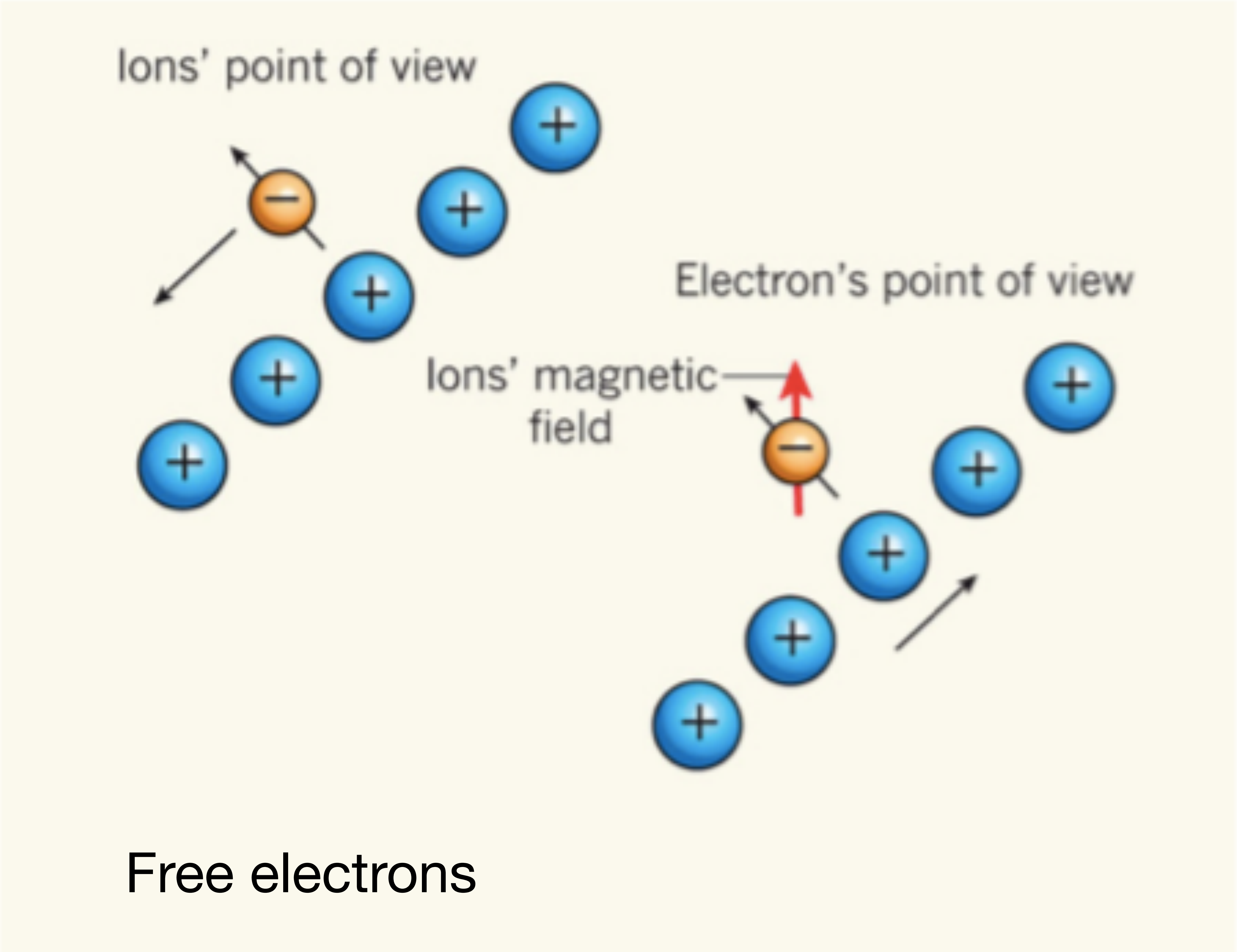
Source: PhysRevLett.92.027003



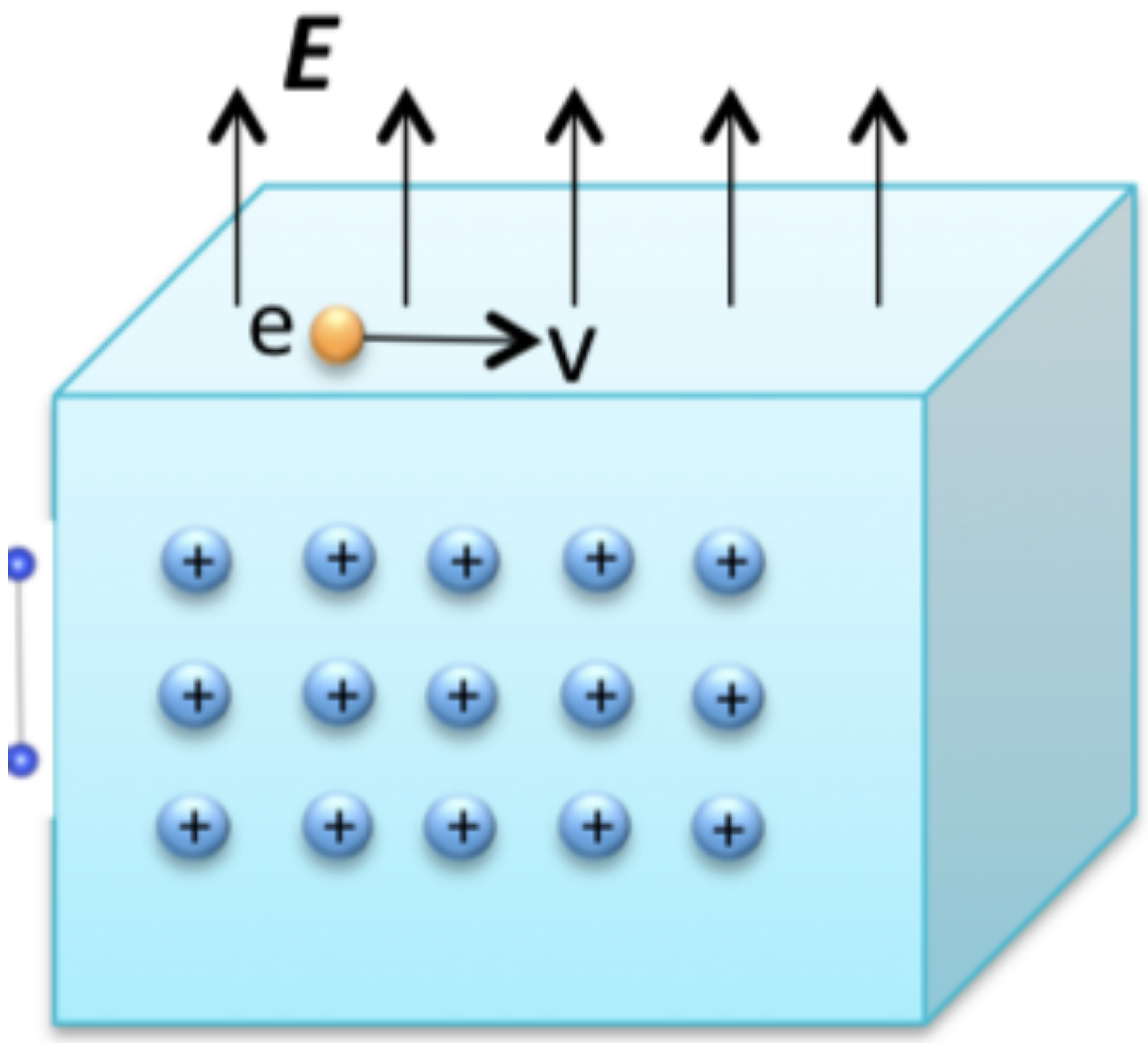
Source:http://www.vlt.phys.tohoku.ac.jp/ MagneticSuperconductivity.html

Spin orbit coupling





Source: Both taken from https://tms16.sciencesconf.org/data/pages/SOC_lecture1.pdf



$$\hat{H}_R = \frac{k^2}{2m} + \alpha \hat{\mathbf{n}} \bullet (\vec{\sigma} \times \mathbf{k}) = \frac{k^2}{2m} + \alpha\left(\sigma^x k_y - \sigma^y k_x\right)$$

$$t \to -t : \ \mathbf{k} \to -\mathbf{k}, \sigma \to -\sigma$$

$$H_R = \begin{pmatrix} k^2/2m & \alpha\left(k_y + ik_x\right) \\ \alpha\left(k_y - ik_x\right) & k^2/2m \end{pmatrix} \Rightarrow \varepsilon_\pm = \frac{k^2}{2m} \pm \alpha k$$

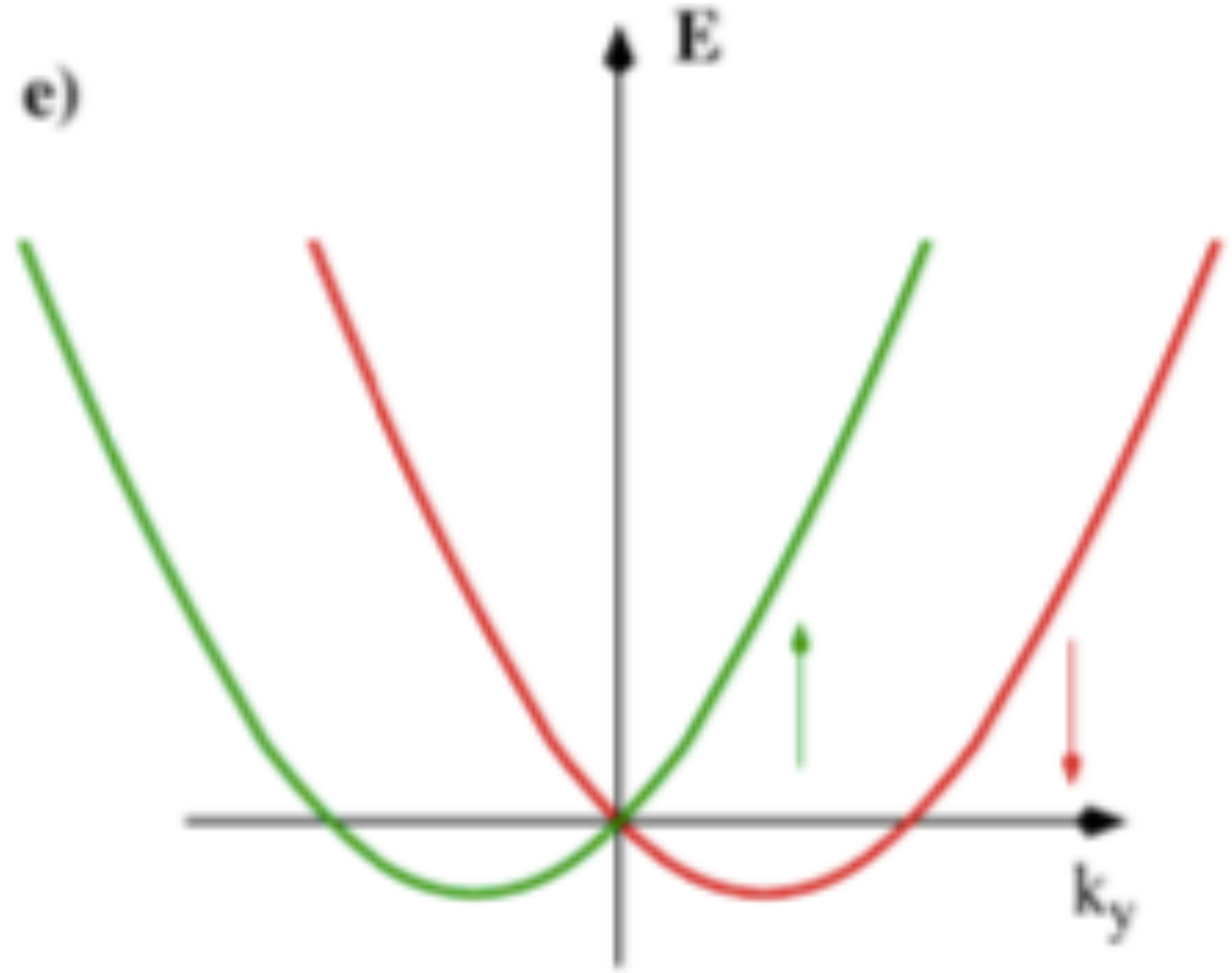
TR - Yes, IR - **Antisymmetric**

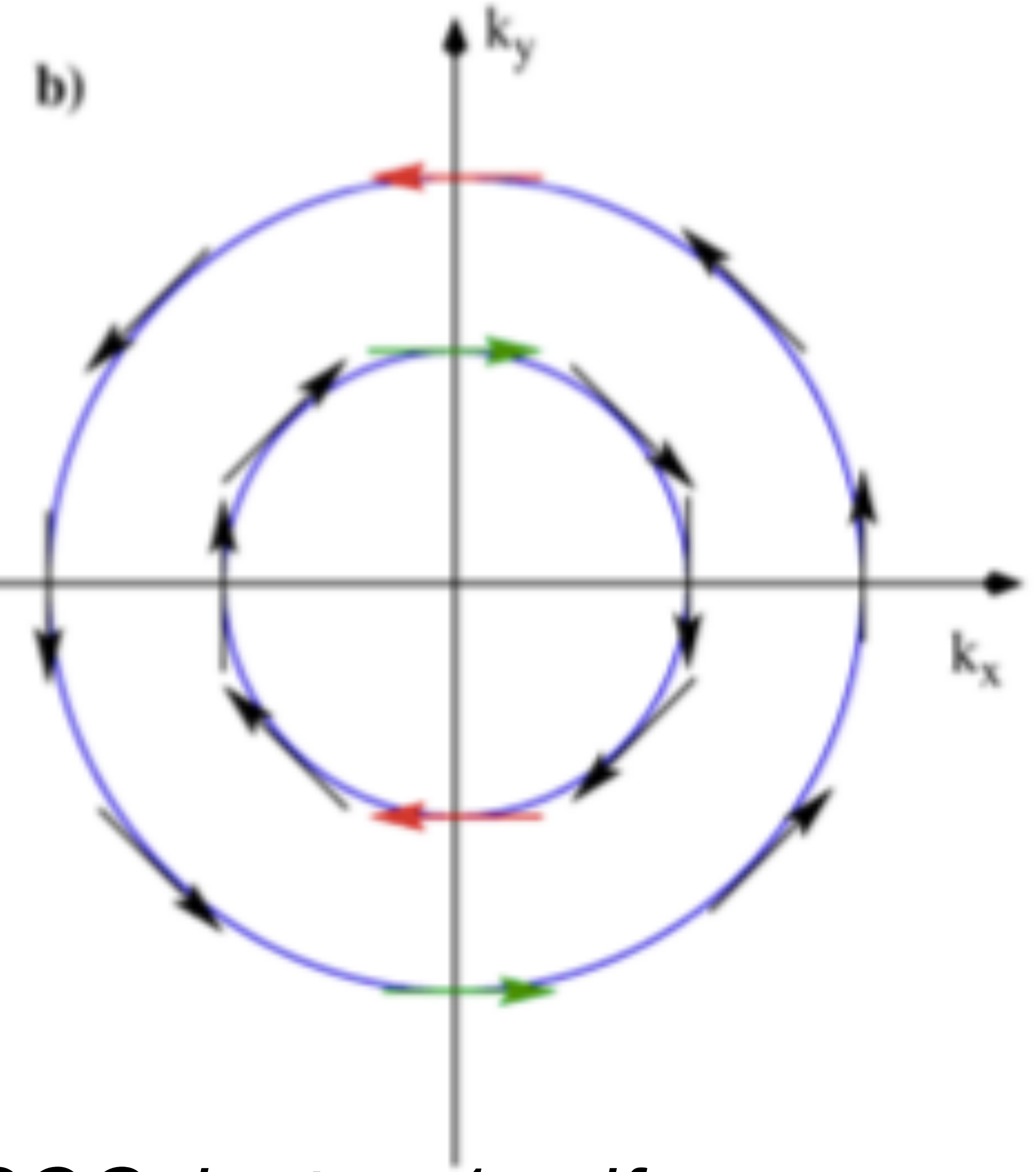
$$H_E = -E_0 z,$$

$$\boxed{\mathbf{B} = \frac{1}{c^2}\mathbf{E} \times \mathbf{v}.}$$

$$H_{SO} = \frac{g\mu_B}{2c^2}(\mathbf{v} \times \mathbf{E}) \cdot \sigma,$$

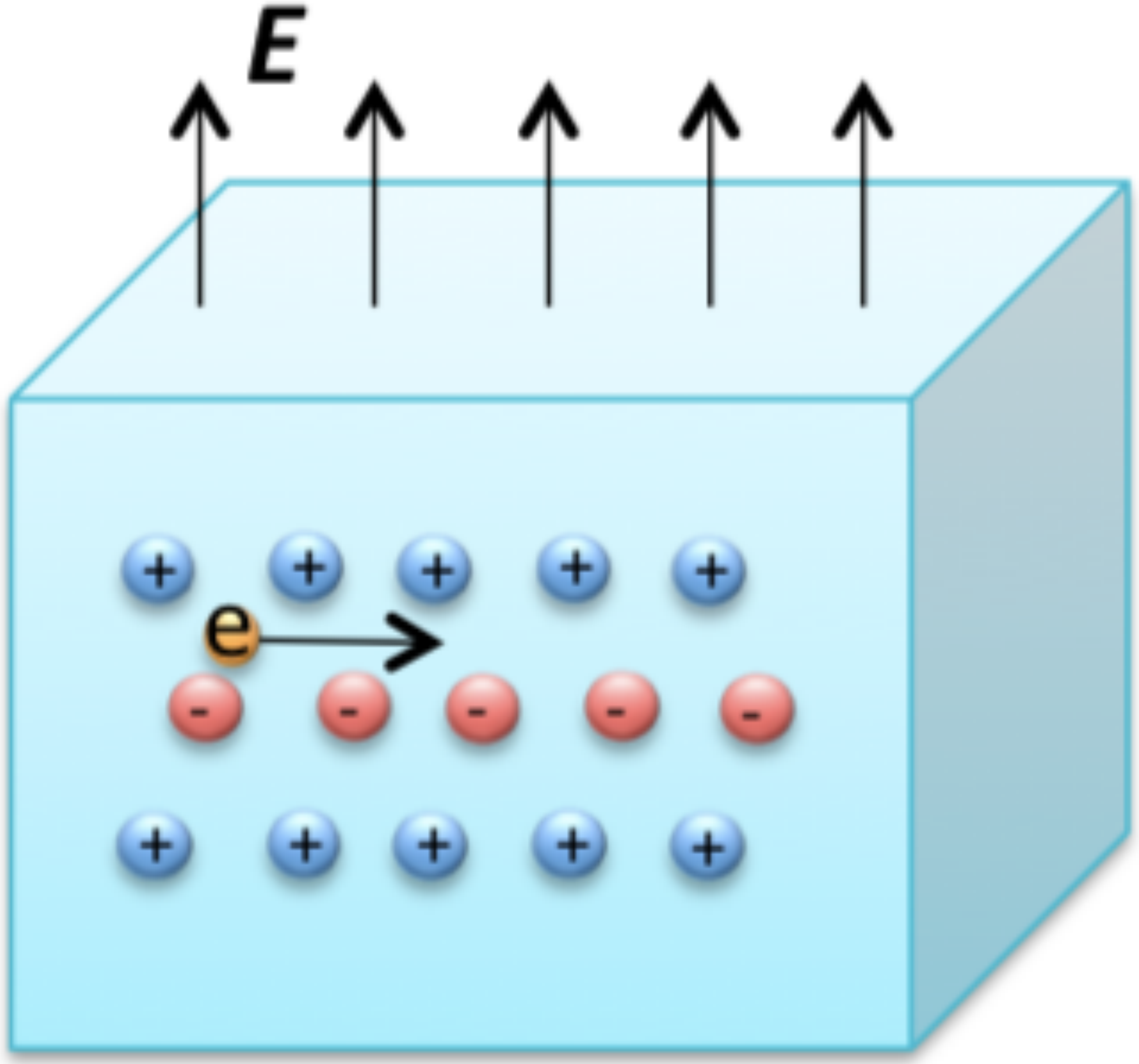
$$H_R = \alpha(\boldsymbol{\sigma} \times \mathbf{p}) \cdot \hat{z},$$





Source: https://tms16.sciencesconf.org/data/pages/SOC_lecture1.pdf

$$\hat{H} = \underbrace{\frac{\hat{p}^2}{2m} + eV}_{\text{non-relativistic}} + \underbrace{\frac{\hat{p}^4}{8m^2c^2}}_{\text{K.E. correction}} + \underbrace{\frac{\hbar^2}{8m^2c^2}\nabla^2 V}_{\text{Darwin term}} + \underbrace{\frac{\hbar}{4m^2c^2}\vec{\sigma}\cdot(\vec{\nabla} V \times \hat{\mathbf{p}})}_{\text{SOI}}$$



Source: https://tms16.sciencesconf.org/data/pages/SOC_lecture1.pdf

- Bulk asymmetry can also induce a SOC term.
- Exact nature depends strongly on the symmetry of the crystal.

Examples:

Cubic: $H_{ASOC} : \ \alpha_{xx}(k_x\sigma_x + k_y\sigma_y + k_z\sigma_z)$

$D_3 : \quad H_{ASOC} : \alpha_{xx}(k_x\sigma_x + k_y\sigma_y) + \alpha_{zz}k_z\sigma_z$

Source: arXiv:1609.05953

Paper Summary:

“Spiral Magnetic field and bound states of vortices in NCS”

Albert Samoilenka, Egor Babaev - PhysRevB. 102.184517

Take the BCS hamiltonian and add a

1. Space dependent B : $\vec{k} \to \vec{k} - q\vec{A} \; + \; - \mu_B \vec{B}(\vec{x})$

2. Spin orbit interaction : $\sum_{\vec{k},\alpha,\beta} \vec{\gamma}(\vec{k}) \cdot \vec{\sigma}_{\alpha,\beta} \to \sum_{\vec{k},\alpha,\beta} \vec{\gamma}(\vec{k} - q\vec{A}) \cdot \vec{\sigma}_{\alpha,\beta}$

$$\boxed{H_{NCS} = \sum_{\vec{x},\alpha} E\left(-i\nabla - q\vec{A}\right)\psi_\alpha^\dagger\left(\vec{x},\tau\right)\psi_\alpha\left(\vec{x},\tau\right) - V\psi_\uparrow^\dagger\psi_\downarrow^\dagger\psi_\downarrow\psi_\uparrow + \sum_{\alpha,\beta,\vec{x}} \psi_\alpha(\vec{x})^\dagger\left(\vec{h} \cdot \vec{\sigma}_{\alpha\beta}\right)\psi_\beta(\vec{x})}$$

$$V > 0 \qquad \vec{h} = \vec{\gamma}\left(-i\nabla - e\vec{A}\right) - \mu_B\vec{B}$$

Now derive GL functional starting from this H

For O (cubic)/T(tetrahedral)

$$\vec{\gamma}(\overrightarrow{k}) = \boldsymbol{\gamma}_0 \overrightarrow{k}$$

Also assume that

$$\mu \gg \omega_D \gg T_c$$
$$\gamma_0 k_F \gg \omega_D \gg \mu_B B$$

Typical Values :

Compound	Structure	T_c (K)	γ (mJ/mol K^2)	H_{c2} (T)	$1/T_1(T)$	KS	$C(T,H)$	TRSB	$\lambda(T)$	E_{ASOC} (meV)	$E_{ASOC}/k_B T_c$
$CePt_3Si$	$P4mm$	0.75	390	2.7$\parallel c$, 3.2$\parallel a$	L	C	L		L	200[9]	3095
$LaPt_3Si$		0.6	11	Type I[10,11]	F		F1	N	F1	200	3868
$CeRhSi_3$	$I4mm$	1.05	110	$\sim 30 \parallel c$, $7 \parallel a$						10	111
$CeIrSi_3$		1.6	100	$\sim 45 \parallel c$, $9.5 \parallel a$	L	C,R				4	29
$CeCoGe_3$		0.64	32	$> 20 \parallel c$, $3.1 \parallel a$						9[12,13]	163
$CeIrGe_3$		1.5	80	$> 10 \parallel c$							
UIr	$P2_1$	0.13	49	0.026							
Li_2Pd_3B	$P4_332$	7	9	2	F	R	F		F2	30	50
Li_2Pt_3B		2.7	7	5	L	C	F/L		L2	200	860
Mo_2Al_3C		9	17.8	15	P			N	F1		
Y_2C_3	$I\bar{4}3d$	18	6.3	30	F2	R	F		L/F2	15	10
La_2C_3		13	10.6	19		C	F1		F2	30	33
$K_2Cr_3As_3$	$P\bar{6}m2$	6.1	70-75	23 $\parallel$, 37$\perp$					L	60	114
$Rb_2Cr_3As_3$		4.8	55	20	P						
$Cs_2Cr_3As_3$		2.2	39	6.5							
BiPd	$P2_1$	3.8	4	0.8	F1		F1		F2	50	153
Re_6Zr	$I\bar{4}3m$	6.75	26	12.2				Y	F1		
Re_3W		7.8	15.9	12.5			F1	N	F1		
Nb_xRe_{1-x}		3.5-8.8	3-4.8	6-15	F	R	F1/2		F1		
$Re_{24}Ti_5$		5.8	111.8	10.75			F1				
$Mg_{10+x}Ir_{19}B_{16-y}$	$I\bar{4}3m$	2.5-5.7	52.6	0.8	F1	R	F1		F1/2		
$Ba(Pt,Pd)Si_3$	$I4mm$	2.3-2.8	4.9-5.7	0.05-0.10			F1				
$La(Rh,Pt Pd,Ir)Si_3$		0.7-2.7	4.4-6	Type I/0.053			F1	N	F1	17(Rh)	93(Rh)
$Ca(Pt,Ir)Si_3$		2.3-3.6	4.0-5.8	0.15-0.27			F1	N			
$Sr(Ni,Pd,Pt)Si_3$		1.0-3.0	3.9-5.3	0.039-0.174			F1				
$Sr(Pd,Pt)Ge_3$		1.0-1.5	4.0-5.0	0.03-0.05			F1				

Table of some known NCS materials.

Source: arXiv: 1609.05953

Given H, we compute the partition function as

$$Z = \int D[\psi, \bar{\psi}] e^{-S[\bar{\psi},\psi]}$$

$$S = \int_0^\beta d\tau d\overrightarrow{x} \sum_{\alpha,\beta=\downarrow\uparrow} a_\alpha^\dagger (\mathbf{h} \cdot \boldsymbol{\sigma}_{\alpha\beta}) a_\beta - V a_\uparrow^\dagger a_\downarrow^\dagger a_\downarrow a_\uparrow$$

$$\mathbf{h} = \left(\partial_T + E - \mu, \overrightarrow{h}\right) \qquad \boldsymbol{\sigma}_{\alpha\beta} = \left(\delta_{\alpha\beta}, \overrightarrow{\sigma_{\alpha\beta}}\right) \qquad \overrightarrow{h} = \overrightarrow{\gamma}\left(-i\nabla - e\overrightarrow{A}\left(\overrightarrow{x}\right)\right) - \mu_B \overrightarrow{B}\left(\overrightarrow{x}\right)$$

$$Z = \int D[\Delta^\dagger, \Delta] e^{\frac{1}{2} ln\ detH - \int d\overrightarrow{x} d\tau \frac{\Delta^\dagger \Delta}{V}} \qquad F = \int d\overrightarrow{x}\, \frac{\Delta^2}{V} - \frac{T}{2} Tr\ ln\ H$$

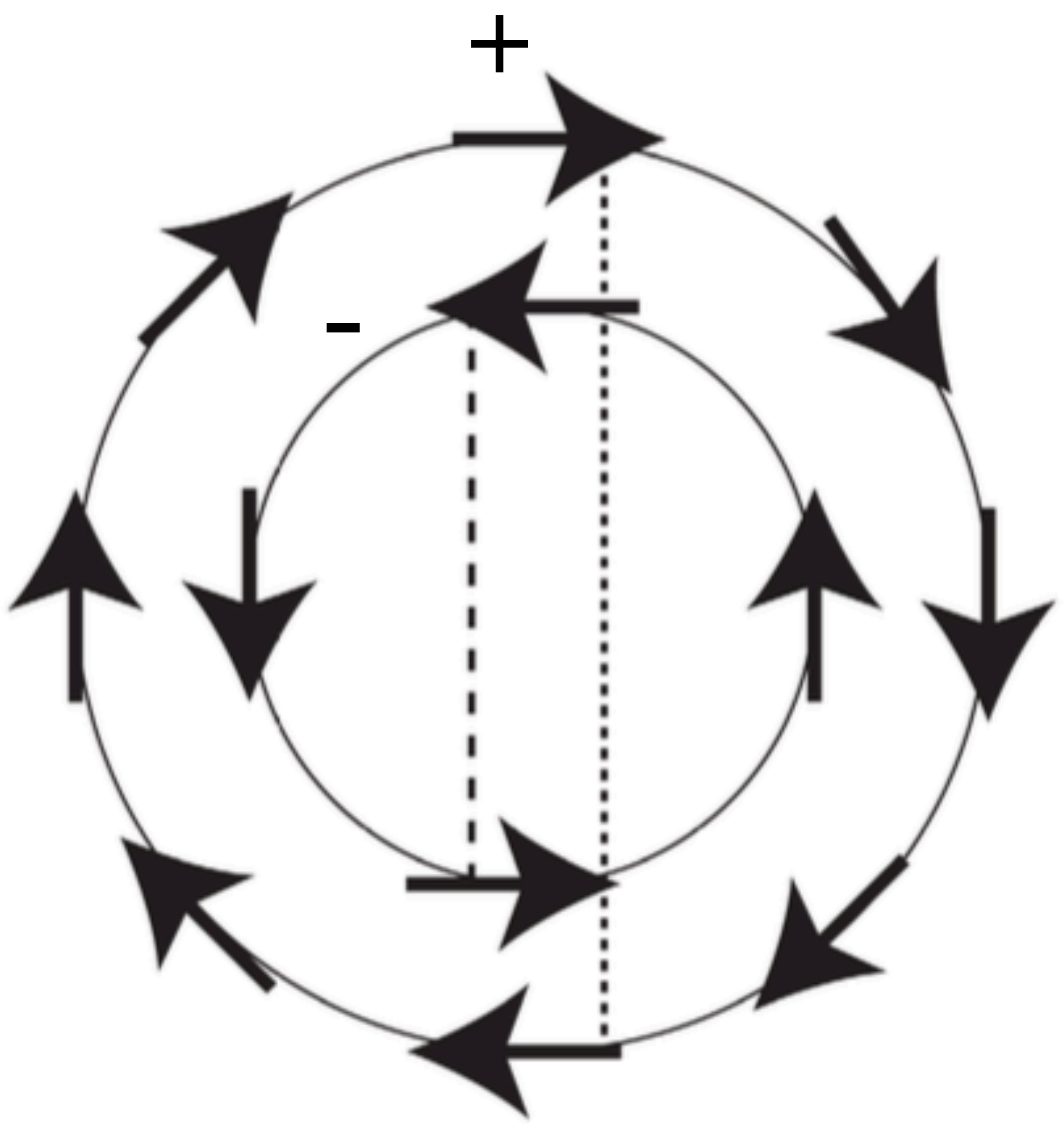
Now expand tr ln H in terms of delta to get F

Final result

$$F_{ncs} = \int d^3\vec{r}\left[\alpha\left|\psi\right|^2 + \sum_{a=\pm 1} K_a \left|\left(v_{aF}D* - 2a\mu_B\vec{B}\right)\psi\right|^2 + \beta\left|\psi\right|^4\right] + \frac{1}{2}(\vec{B} - \vec{H})^2$$

$$\alpha = N\, ln(\frac{T}{T_C}) \qquad K_a \propto \nu(E_{aF}),$$

$$T_c = \frac{2e^{\gamma_{\text{Euler}}}}{\pi}\omega_D e^{-\frac{1}{NV}} \qquad \beta = \frac{7\zeta(3)}{(4\pi T)^2}N$$



Taken from arXiv:1609.05953

Well defined functional: manifestly bounded from below

Rescaled GL

$$F = \int d\overrightarrow{r} \left[\frac{\left(\overrightarrow{B} - \overrightarrow{H}\right)^2}{2} + \sum_{a=\pm 1} \frac{\left|\mathscr{D}_a \Psi\right|^2}{2\kappa_c} - \left|\Psi\right|^2 + \frac{\left|\Psi\right|^4}{2} \right]$$

$$\psi_{old} = \sqrt{\frac{-\alpha}{2\beta}} \psi_{new}$$

$$\mathscr{D}_a \;=\; i\nabla - \overrightarrow{A} - (\gamma + a\nu)\overrightarrow{B}$$

$$\gamma = \sqrt{-\alpha} \left(\sum_{a=\pm 1} a K_a v_{aF} \right) 2\mu_B \kappa_c \left(\frac{2e^2}{\beta} \right)^{\frac{3}{4}}$$

$$\nu = \sqrt{-\alpha K_+ K_-} \left(\sum_{a=\pm 1} v_{aF} \right) 2\mu_B \kappa_c \left(\frac{2e^2}{\beta} \right)^{\frac{3}{4}}$$

$$\gamma \propto \gamma_0, \;\; for \; \gamma_0 k_F \ll \mu$$

GL equations:

$$\sum_a \frac{D_a^2\psi}{2\kappa_c} - \psi + \psi|\psi|^2 = 0,\ c.c = 0$$

$$\nabla \times \left[\vec{B} - \vec{H} - \sum_{a=\pm 1} (\gamma + a\nu)\vec{J}_a\right] = \sum_a \vec{J}_a$$

$$\vec{J}_a = \frac{Re(\psi^* \mathscr{D}_a \psi)}{\kappa_c}$$

Magnetic Field configuration is given by

$$\vec{B} = Re(\vec{w}) \qquad \vec{w} = \eta f\,\hat{k} - \hat{k} \times \nabla f \qquad \nabla^2 f + \eta^2 f = 0^*$$

$$\eta = \eta_1 + i\eta_2 \ = \ \frac{-\gamma + i\chi}{\gamma^2 + \chi^2},\ \chi = \sqrt{\frac{\kappa_c}{2} + \nu^2}$$

Spiral Meissner Effect

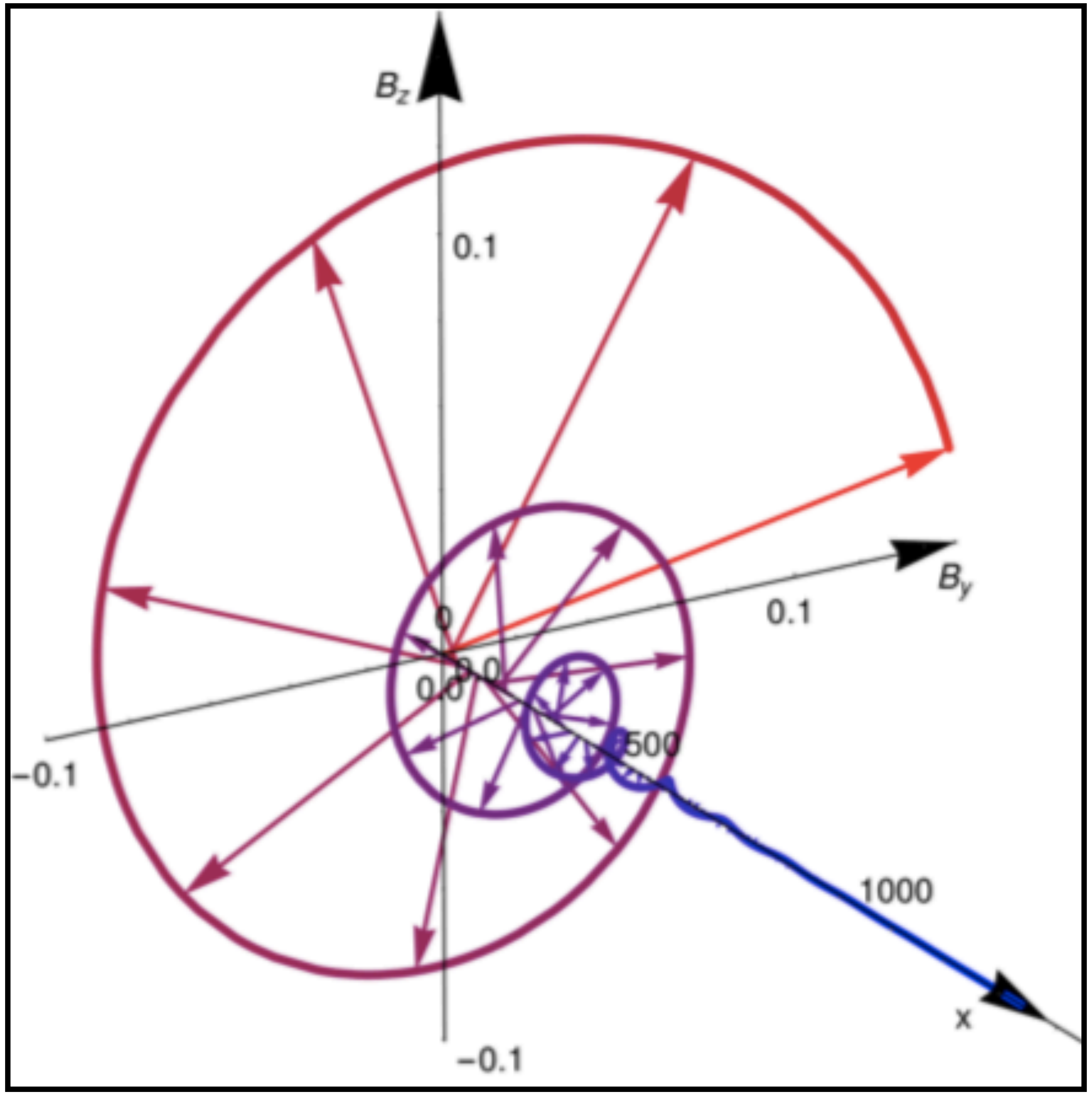
GL result shows that

$$f \propto e^{i\eta x}$$

$$\tilde{B} = B_z + iB_y = -\frac{i\eta\kappa_c}{2\chi}(H_z + iH_y)e^{i\eta x} \propto e^{-\eta_2 x + i\eta_1 x}$$

$\eta_1 \propto \gamma \; (\propto \gamma_0)$ Determines handedness

$$\lambda = \frac{1}{\eta_2}$$



Taken from arXiv:2003.10918v1

Vortex State

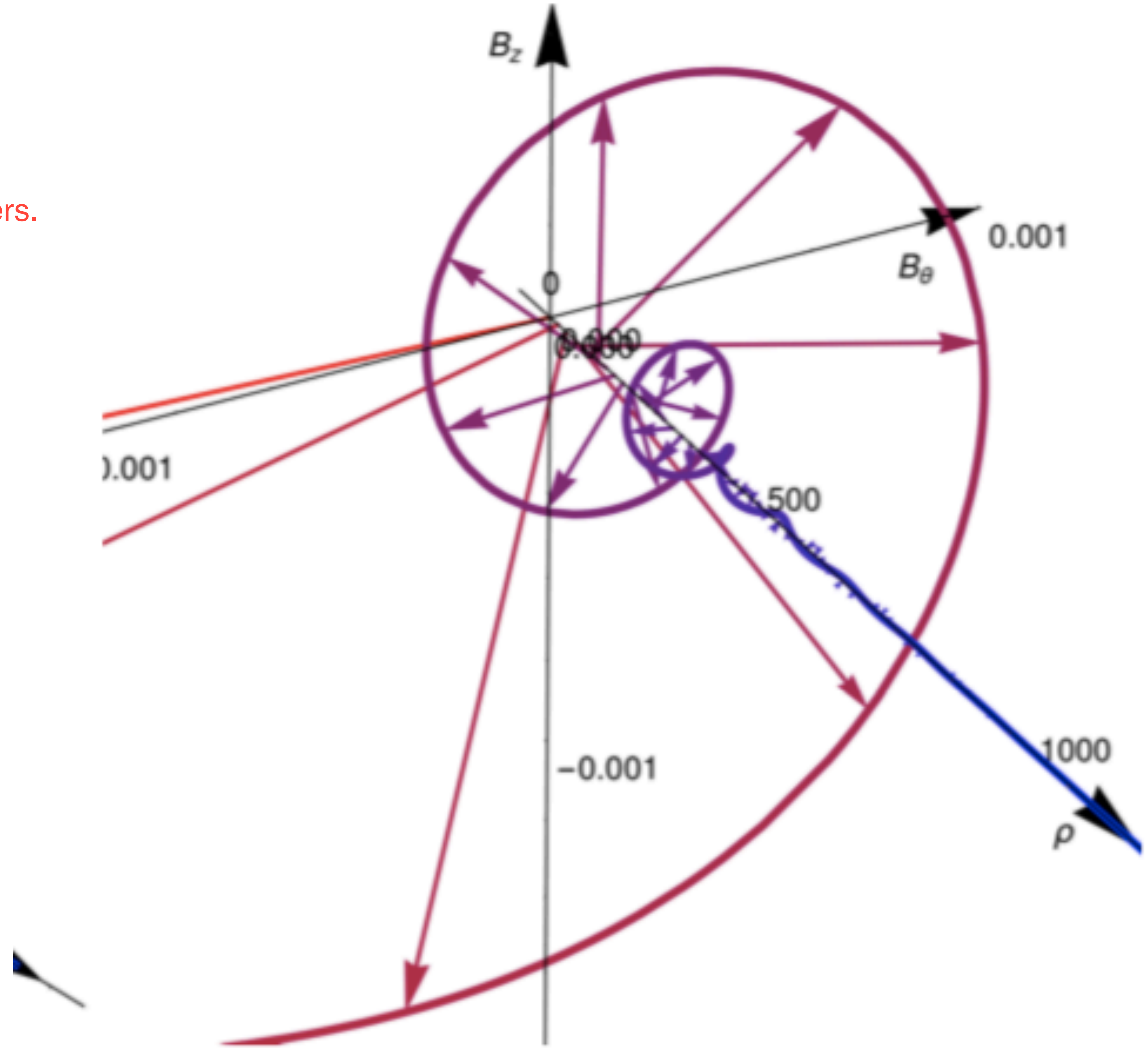
Using the equations as before, we can compute structure of vortex magnetic field and Hc1. Work in the London limit to do this.

$$f = \frac{i\pi}{2}\eta n H_0^{(1)}(\eta\rho)$$

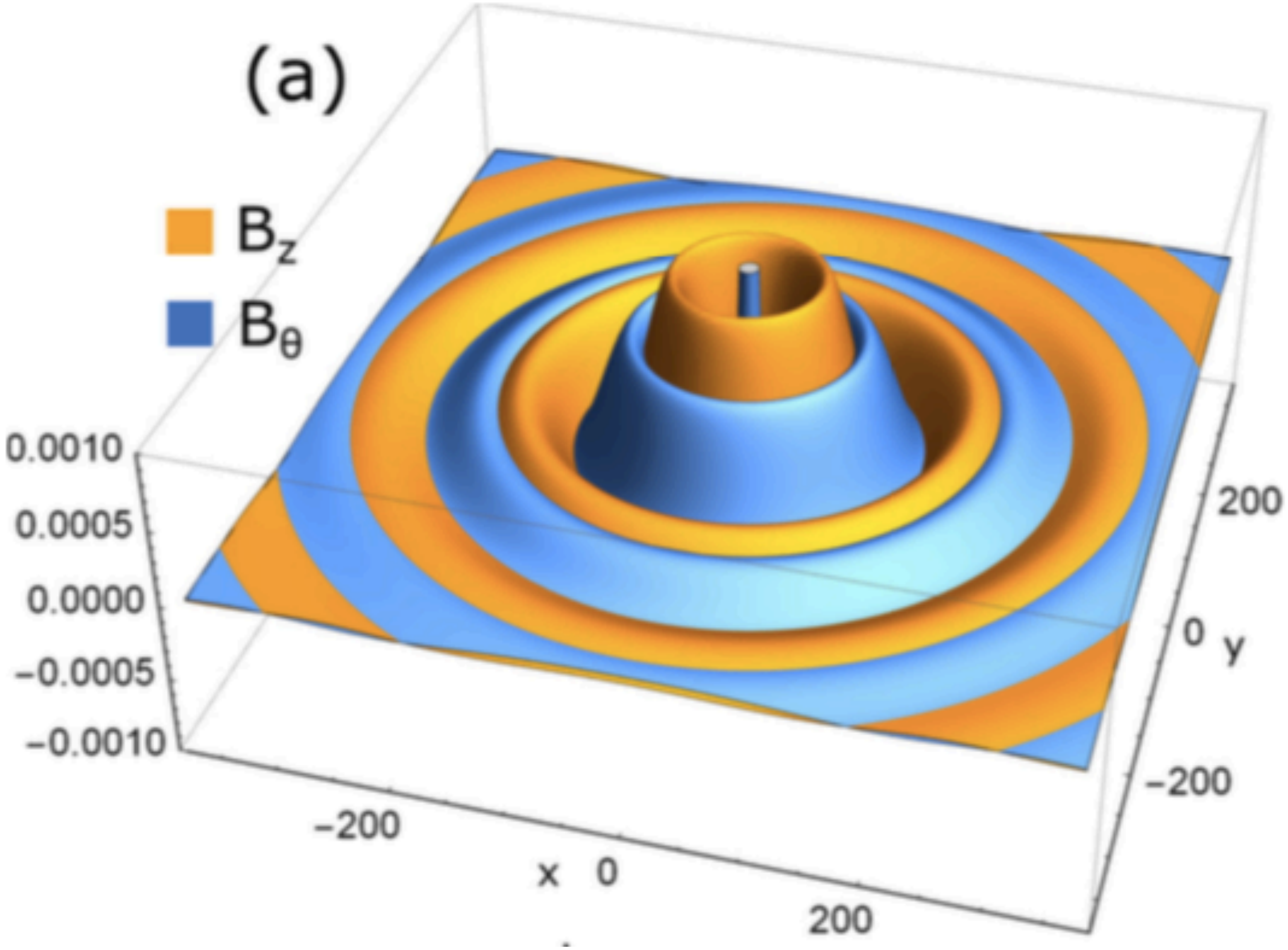
This part was missed by previous papers.

$$\overrightarrow{B}(\rho,\theta,z) = Re\left[\frac{i\pi}{2}n\eta^2\left(0, H_1^{(1)}, H_0^{(1)}\right)\right]$$

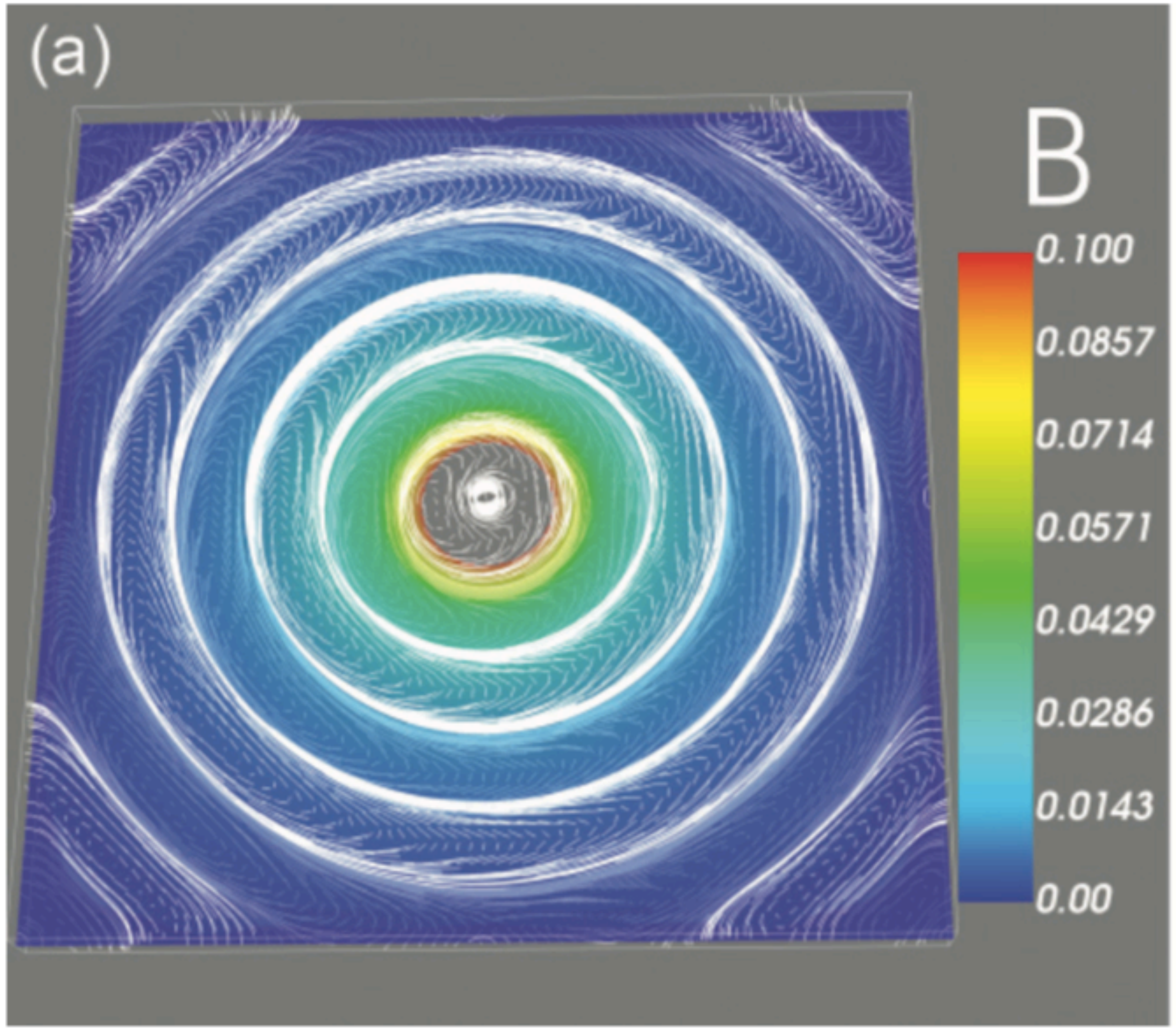
$$\boxed{\tilde{B} = B_z + iB_\theta \propto \frac{e^{i\eta\rho}}{\sqrt{\rho}}}$$



Taken from arXiv:2003.10918v1



Plot of B vs (x,y) for the vortex states.



Plot of |B| vs (x,y) in 2D

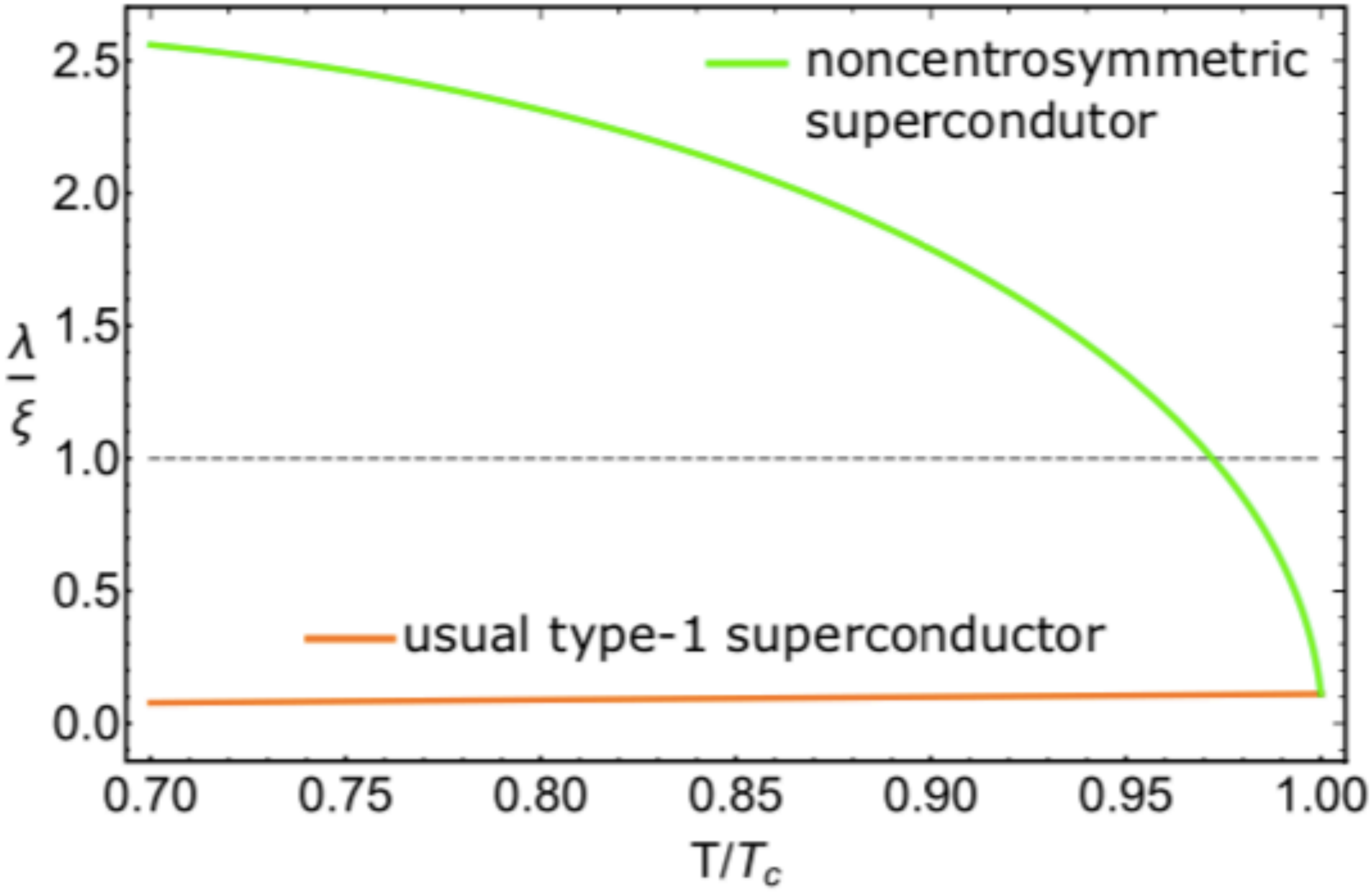
Both taken from PhysRevB 102.184517

Crossover to Type I

Given the coherence length and the penetration depth, we calculate the GL parameter

$$\kappa = \frac{\lambda}{\xi} = \frac{\sqrt{2\kappa_c}}{\eta_2} = \kappa_c \frac{1 + \frac{2}{\kappa_c}(\gamma^2 + \nu^2)}{\sqrt{1 + \frac{2\nu^2}{\kappa_c}}}$$

- Near Tc, $\gamma, \nu \to 0, so\ \kappa = \kappa_c$
- As T is lowered, κ increases.

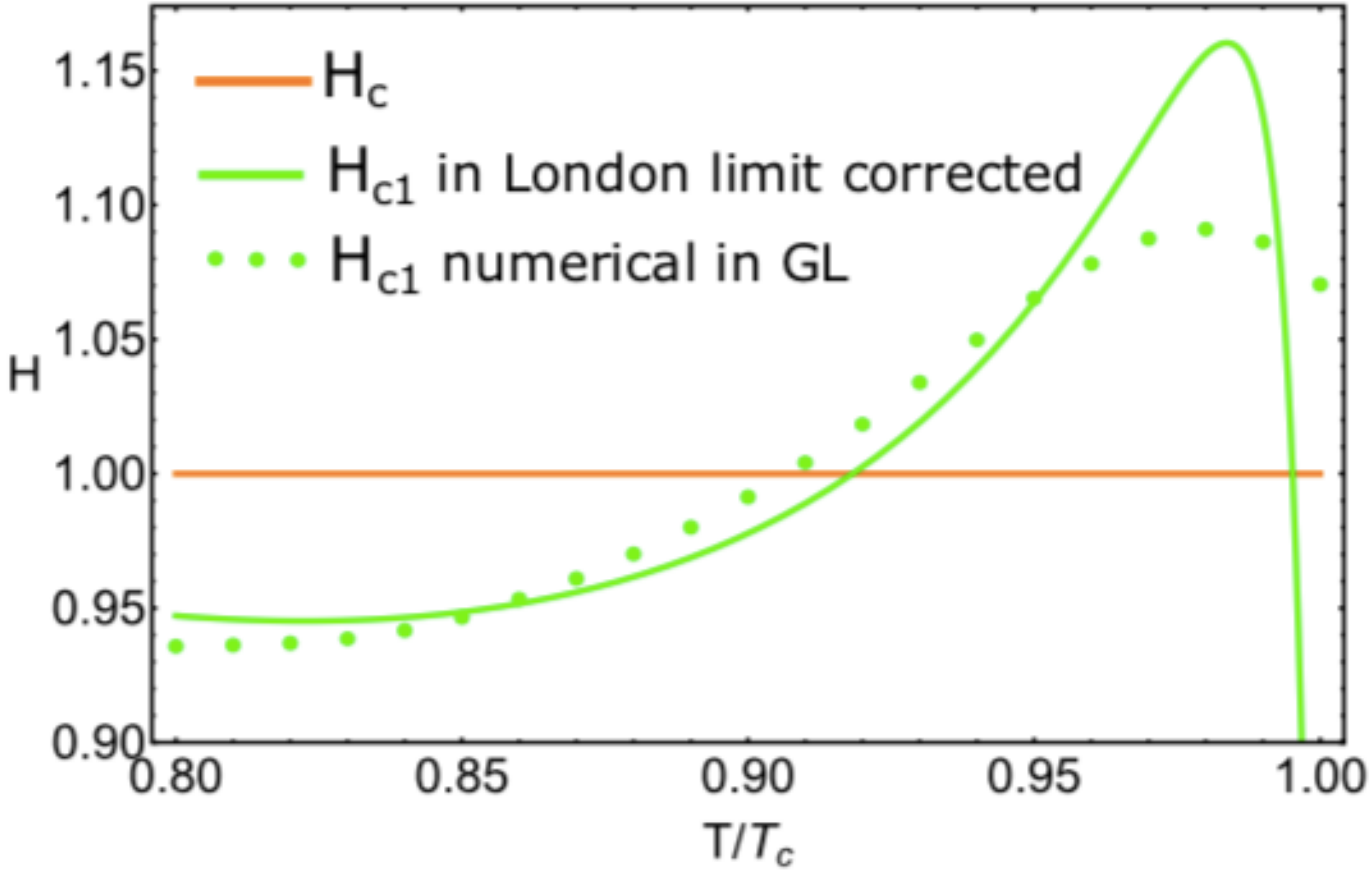


Taken from PhysRevB 102.184517

Now we look at energy of a vortex configuration. We find the free energy for the vortex configuration to be

$$F_v = 2\pi n(nH_{c1}^L + H)$$

$$H_{c1}^L = \frac{\chi}{\kappa_c}\left[\eta_1 tan^{-1}(\frac{\eta_1}{\eta_2}) + \eta_2 ln\frac{2e^{-\gamma_{euler}}}{\xi}\right]$$



Taken from PhysRevB 102.184517

Inter-vortex Interaction : Bound states

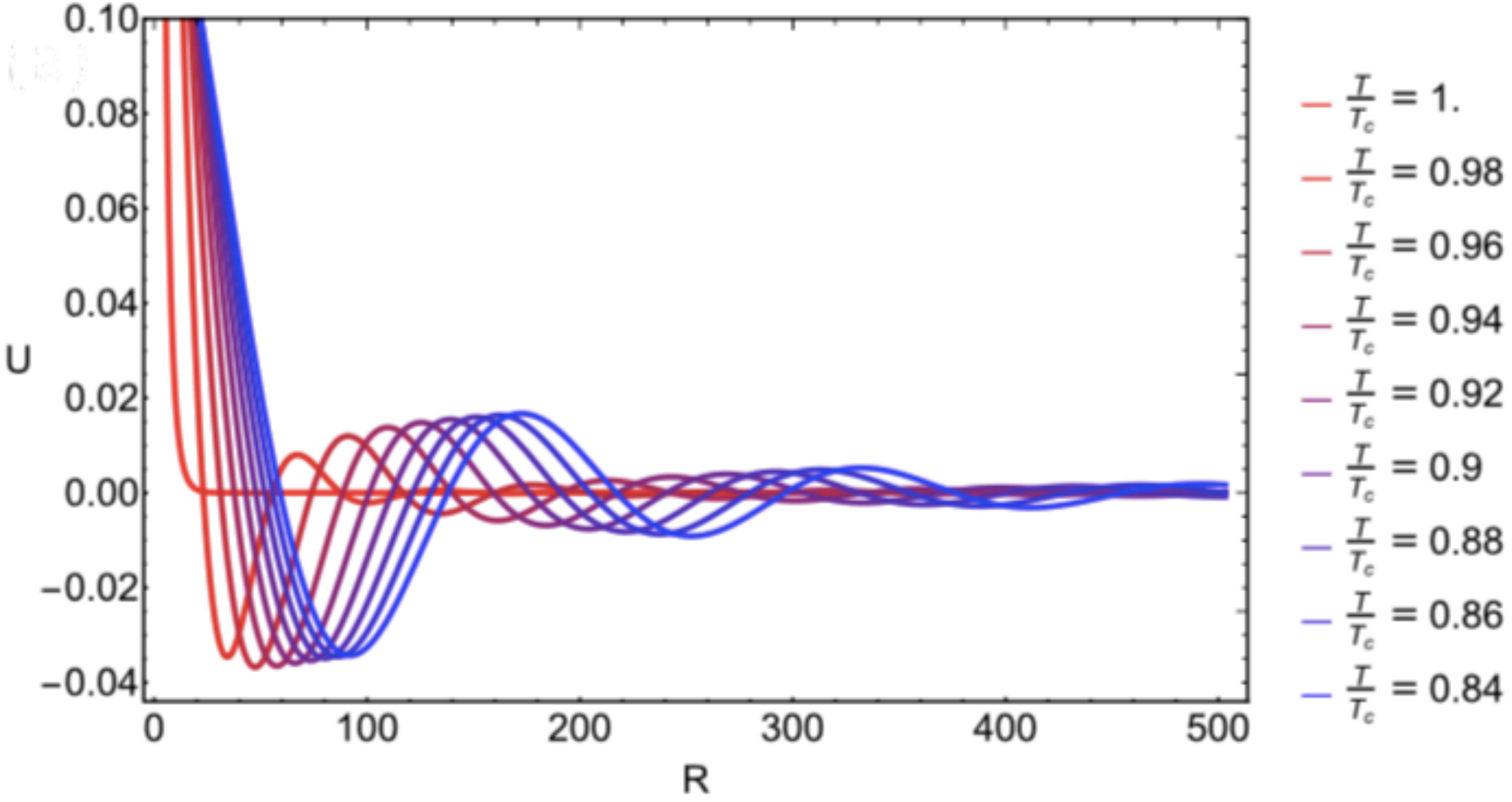
$$U(R) \propto n_1 n_2 e^{-\eta_2 R} cos(\eta_1 R + \phi_0)$$

This too was
missed by previous
works

- Non-monotonic inter-vortex

Interaction.

- Vortices can form pairs.
- Many vortex pairs



Taken from PhysRevB 102.184517

Conclusion: Standard GL vs NCS

Property	Standard GL	NCS
Meissner Effect	Normal Decay	Spiral Decay
Vortex Magnetic Field	Normal Decay	Spiral Decay
Intervortex interaction	Monotonic	Non-Monotonic
Crossover	Doesn’t generally occur	Can occur

Fluctuations in superconductors

Example

$$F_{GL} = \int a|\Psi|^2 + \frac{b}{2}|\Psi|^4 + \frac{1}{4m}|\nabla\Psi|^2$$

Minimizing the free energy functional we have

$$|\widetilde{\Psi}|^2 = \begin{cases} -\alpha T_c \epsilon / b, & \epsilon < 0 \\ 0, & \epsilon > 0 \end{cases}$$

$$F = (\mathcal{F}[\Psi])_{\min} = \mathcal{F}[\widetilde{\Psi}] = \begin{cases} F_N - \dfrac{\alpha^2 T_c^2 \epsilon^2}{2b} V, & \epsilon < 0 \\ F_N, & \epsilon > 0 \end{cases}$$

Source: arXiv: cond-mat/0109177v1

$$\boxed{\Psi = \varphi_{mF}\left(= 0\ for\ \epsilon > 0\right) + \psi}$$

Decompose the net field into mean field contribution (can be spatially non-uniform)

And thermal fluctuations.

$$F[\Psi] \equiv F[\psi] = \int a\,|\psi|^2 + \frac{b}{2}\,|\psi|^4 + \frac{1}{4m}\,|\nabla\psi|^2$$

Source: arXiv: cond-mat/0109177v1

$$F[\Psi_{\mathbf{k}}] = F_N + \sum_{\mathbf{k}} \left[a + \frac{\mathbf{k}^2}{4m}\right] |\Psi_{\mathbf{k}}|^2$$

$$Z = \prod_{\mathbf{k}} \int d^2\Psi_{\mathbf{k}} \exp\left\{-\alpha(\epsilon + \frac{\mathbf{k}^2}{4m\alpha T_c})|\Psi_{\mathbf{k}}|^2\right\} \qquad F(\epsilon > 0) = -T \ln Z = -T \sum_{\mathbf{k}} \ln \frac{\pi}{\alpha\left(\epsilon + \frac{\mathbf{k}^2}{4m\alpha T_c}\right)}.$$

F integral diverges. Reason?

Although F diverges, taking derivatives to calculate observables like specific heat/susceptibility etc. can converge.

$$\delta C_+ = -\frac{1}{VT_c}\left(\frac{\partial^2 F}{\partial \epsilon^2}\right) = \frac{1}{V}\sum_{\mathbf{k}} \frac{1}{\left(\epsilon + \frac{\mathbf{k}^2}{4m\alpha T_c}\right)^2}.$$

Convergent result

$$\delta C_+ = \frac{1}{8\pi}\frac{(4m\alpha T_c)^{1.5}}{\sqrt{\epsilon}}$$

Source: arXiv: cond-mat/0109177v1

Fluctuational Susceptibility

Since fluctuation induced diamagnetism eventually leads to meissner effect as we reduce the temperature, we can expect it to be small wrt diamagnetic susceptibility of a superconductor. However, it can be comparable to the value of diamagnetic/paramagnetic susceptibility of a normal metal.

$$\chi = \left\{ -\frac{e^2 k_F}{12\pi^2 mc^2} \right\} [7\zeta(3)/12]^{1/2} [T_c/(T-T_c)]^{1/2}$$

$$\approx -10^{-7} \times [T_c/(T-T_c)]^{1/2}.$$

Source: Physrev.180.527

The quantity in brackets is diamagnetic susceptibility of free electrons (landau susceptibility) for free electrons.

We reconsider the free energy

$$F = \int d^3\vec{r}\left[\alpha\left|\psi\right|^2 + \sum_{a=\pm1} K_a \left|\left(v_{aF}D^* - 2a\mu_B\vec{B}\right)\psi\right|^2\right] + \frac{1}{2}(\vec{B} - \vec{H})^2$$

Now expanding this for a constant magnetic field along z direction we get

$$F = \int d^3\vec{r}\left[a\left|\psi\right|^2 - \gamma\cdot\vec{j}\cdot\vec{B} + \delta\cdot\left|D\psi\right|^2\right]$$

$$a = \left(\alpha + 4\mu_B^2 B^2 \sum k_a\right) \sim \alpha$$

$$\gamma = 2\mu_B \sum_{a=\pm1} K_a a v_{aF}$$

$$\delta = \sum K_a v_{aF}^2$$

$$\xi^2 = \frac{\delta}{|\alpha|}$$

Free energy reads

$$F = -T \times \frac{SB}{\Phi_0} \sum_{m,k} log \frac{\pi T}{a + 2kB\gamma + \delta .2M \cdot \left[\omega_c \left(m + \frac{1}{2}\right) + \frac{k^2}{2M}\right]}$$

$$\boxed{F = \frac{T\delta V e^2}{12} B^2 * \frac{\pi}{\sqrt{\delta}} \left[\frac{1}{\sqrt{a}} + \frac{1}{2}\frac{B^2\gamma^2}{\delta a^{1.5}} + \ldots.\right] - \frac{TS}{2\delta} \int_{-\infty}^{\infty} \frac{dk}{2\pi} \int_0^{\infty} \log \frac{\pi k_B T}{a + 2kB\gamma + \delta k^2 + z} dz}$$

For small B only

2nd integral is divergent, hence not evaluated at this stage.

NCS result:

$$\chi^{fluc}_{NCS} = \frac{-T\pi\delta V e^2}{6} * \xi_{GL}\left[1 + \frac{3B^2\gamma^2}{\delta a^1} + \ldots.\right] + \frac{TVB^2}{\delta^{3.5}} * \frac{B^2\gamma^4}{\sqrt{a}} + \ldots$$

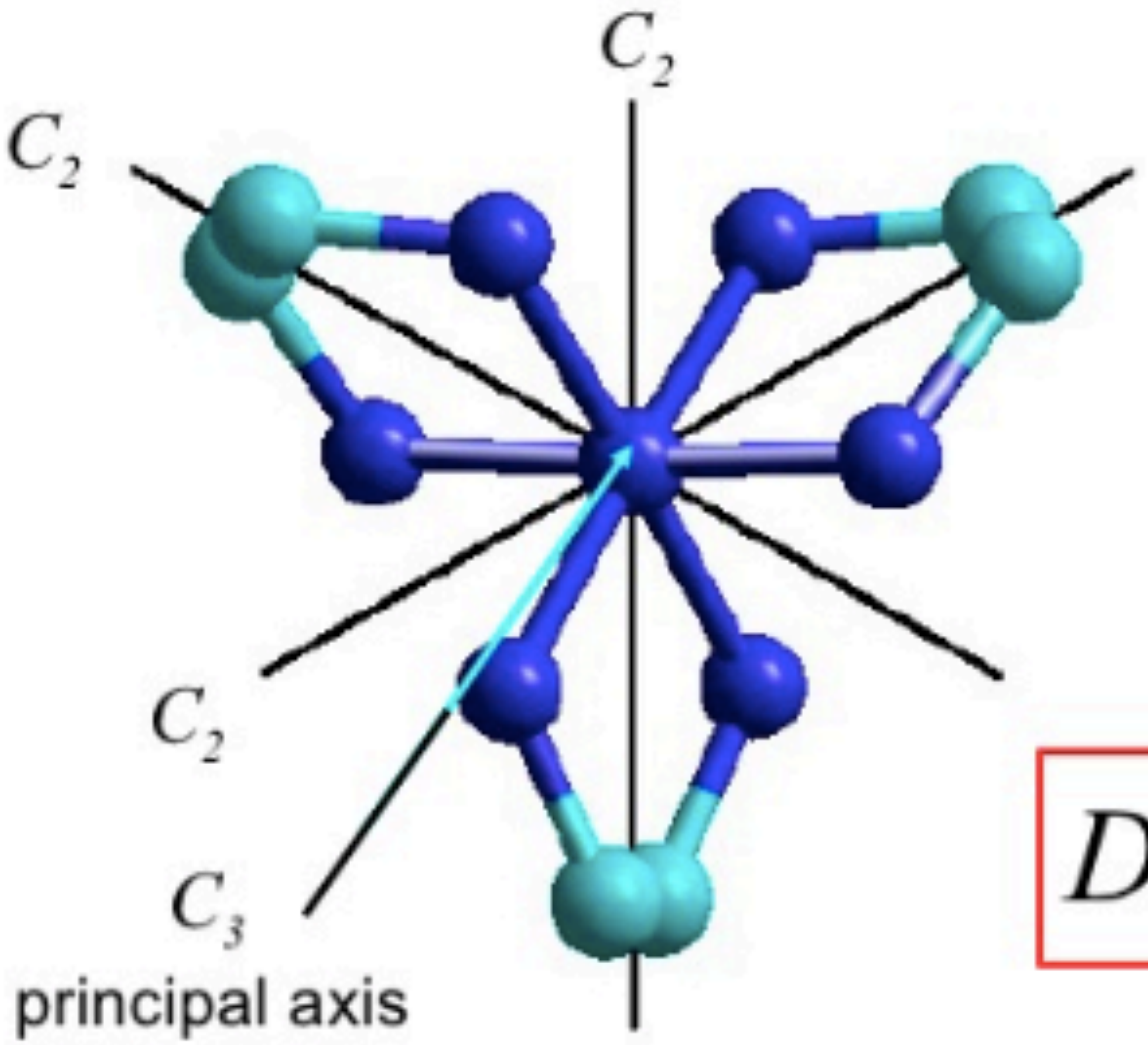
$$\frac{B^2\gamma^2}{\delta} + \frac{\delta Be}{\pi} \ll a$$

BCS result:

$$\chi^{fluc}_{BCS} = -V \times \frac{1}{6\pi}\frac{e^2}{(hc)^2}T\xi_{GL}$$

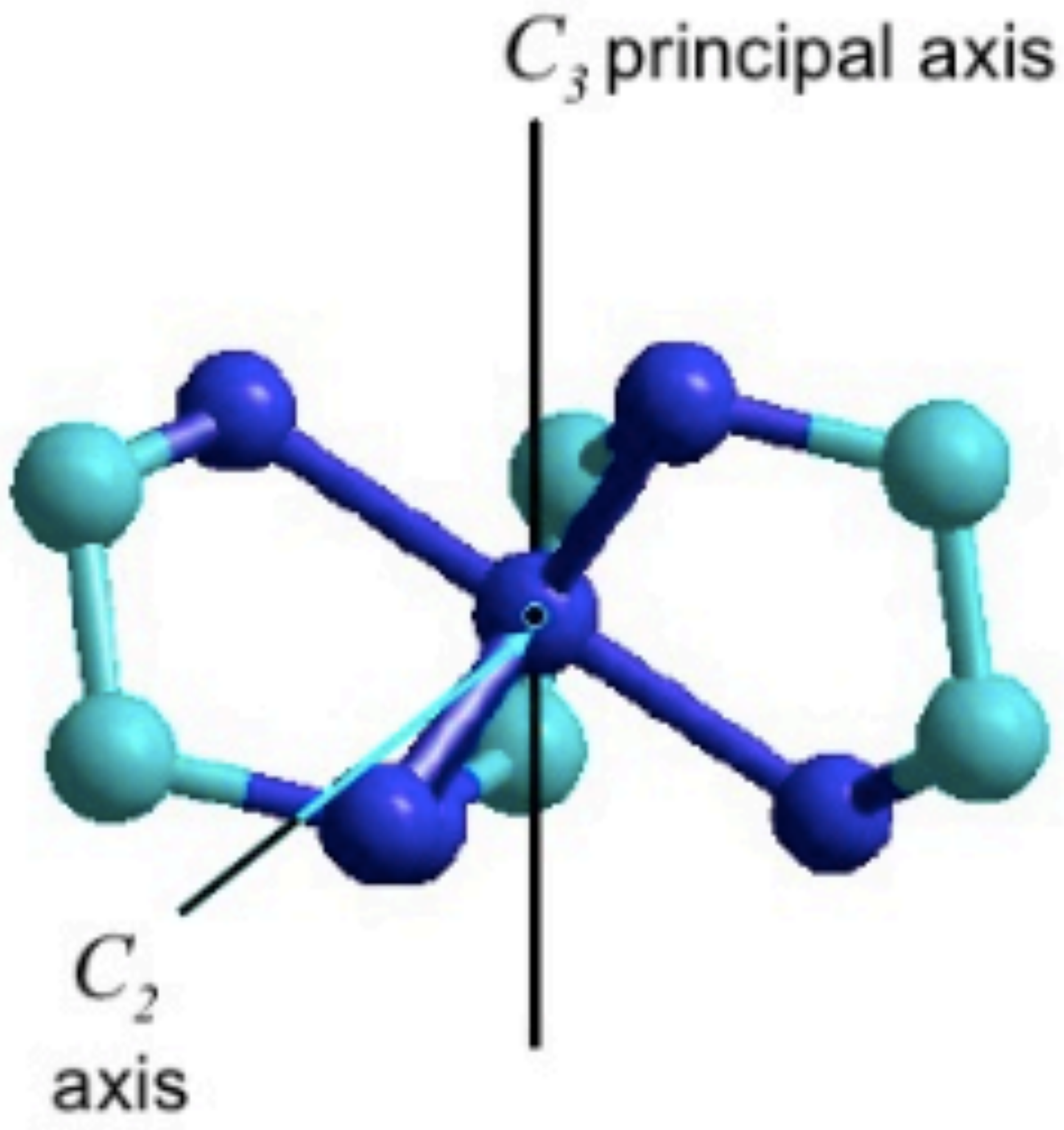
As stated in Physrev.180.527

Thank you



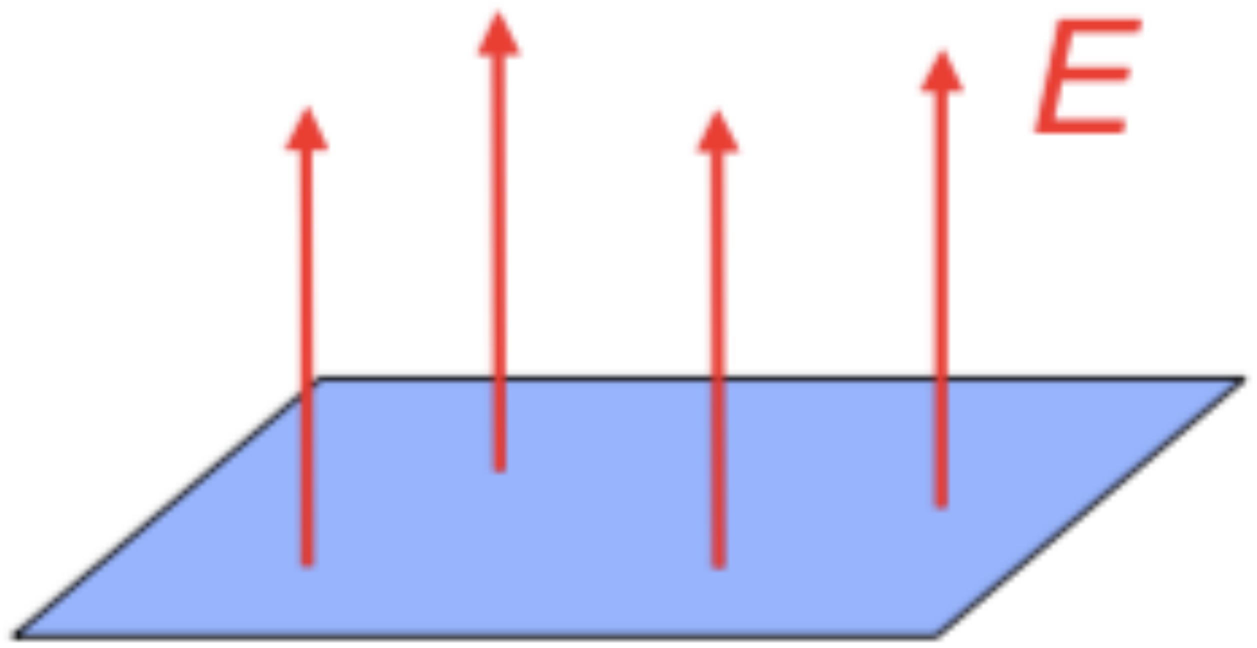
view down the C_3 axis of $[Co(en)_3]^{3+}$ showing the three C_2 axes.

D_3



view down one of the three C_2 axes of $[Co(en)_3]^{3+}$ at right angles to C_3

ASOC



$$\epsilon_{\overrightarrow{k}} = \frac{k^2}{2m}$$

Special relativity yields

$$\vec{B} = -\frac{\vec{v}_{\vec{k}}}{c} \times \vec{E} = \frac{\hbar E}{mc}(\vec{k} \times \hat{z})$$

$$-\mu_B \,.\, \overrightarrow{B} = \frac{h\mu_B E}{mc}(\overrightarrow{k} \times \hat{z}) \,.\, \overrightarrow{S} = \alpha\gamma(\overrightarrow{k}) \,.\, \overrightarrow{S}$$

Antisymmetric SOC

$$\vec{\gamma}(\overrightarrow{-k}) = -\,\vec{\gamma}(\overrightarrow{-k})$$

Given H, we compute the partition function as

$$Z = \int D[\psi, \bar{\psi}] e^{-S[\bar{\psi},\psi]}$$

$$S = \int_0^\beta d\tau d\overrightarrow{x} \sum_{\alpha,\beta=\downarrow\uparrow} a_\alpha^\dagger (\mathbf{h} \cdot \boldsymbol{\sigma}_{\alpha\beta}) a_\beta - V a_\uparrow^\dagger a_\downarrow^\dagger a_\downarrow a_\uparrow$$

$$\mathbf{h} = \left(\partial_T + E - \mu, \overrightarrow{h}\right) \qquad \boldsymbol{\sigma}_{\alpha\beta} = \left(\delta_{\alpha\beta}, \overrightarrow{\sigma_{\alpha\beta}}\right) \qquad \overrightarrow{h} = \overrightarrow{\gamma}\left(-i\nabla - e\overrightarrow{A}\left(\overrightarrow{x}\right)\right) - \mu_B \overrightarrow{B}\left(\overrightarrow{x}\right)$$

Now do mean field decoupling

$$\exp\left[V \int d\overrightarrow{x} d\tau a_\uparrow^\dagger a_\downarrow^\dagger a_\downarrow a_\uparrow\right] = \int D\left[\Delta, \Delta^\dagger\right] exp\big(- \int d\tau d\overrightarrow{x} \left[\frac{\Delta^\dagger \Delta}{V} + \Delta^\dagger a_\downarrow a_\uparrow + \Delta a_\uparrow^\dagger a_\downarrow^\dagger\right]\big)$$

This gives

$$Z = \int D[\Delta^\dagger, \Delta] D[b] e^{-\int d\overrightarrow{x} d\tau (b^T \frac{H}{2} b + \frac{\Delta^\dagger \Delta}{V})} \qquad b = (a_\uparrow, a_\downarrow, a_\downarrow^\dagger, a_\uparrow^\dagger)$$

$$H_0 = \begin{pmatrix} 0 & -h^T \\ h & 0 \end{pmatrix}, \ \Lambda = \begin{pmatrix} \delta^\dagger & 0 \\ 0 & \delta \end{pmatrix} \qquad h = \mathbf{h} \cdot \boldsymbol{\sigma} \qquad \delta = \boldsymbol{\sigma}(0,0,i\Delta,0)$$

Integrating b we get

$$Z = \int D[\Delta^\dagger, \Delta] e^{\frac{1}{2} ln\ det H - \int d\overrightarrow{x} d\tau \frac{\Delta^\dagger \Delta}{V}} \qquad F = \int d\overrightarrow{x}\ \frac{\Delta^2}{V} - \frac{T}{2} Tr\ ln\ H$$

Now expand F in terms ofΔ

$$Tr \log H = Tr \log \left(1 + H_0^{-1} \Lambda\right) = \sum_{\gamma=1}^{\infty} \frac{(-1)^{\nu+1}}{\nu} Tr \left[\left(\hat{g} \hat{\delta} \hat{g}^T \delta^\dagger \right) \right] \qquad \hat{h} \hat{g} = \delta\left(\overrightarrow{x} - \overrightarrow{x'}\right) \delta(\tau - \tau')$$

HS transformation

Boundary conditions in GL

$$\vec{n} \cdot \sum_a D_a \psi = 0$$

$$\vec{n} \times \left[\vec{B} - \vec{H} - \chi_a \vec{J}_a \right] = 0$$

Hankel function of the 1st kind: $H_0^1(\eta\rho) \to const \times ln(\rho)$

For imaginary η, it reduces to Bessel function K_0