# Non-Centrosymmetric Superconductors: Response and Fluctuations

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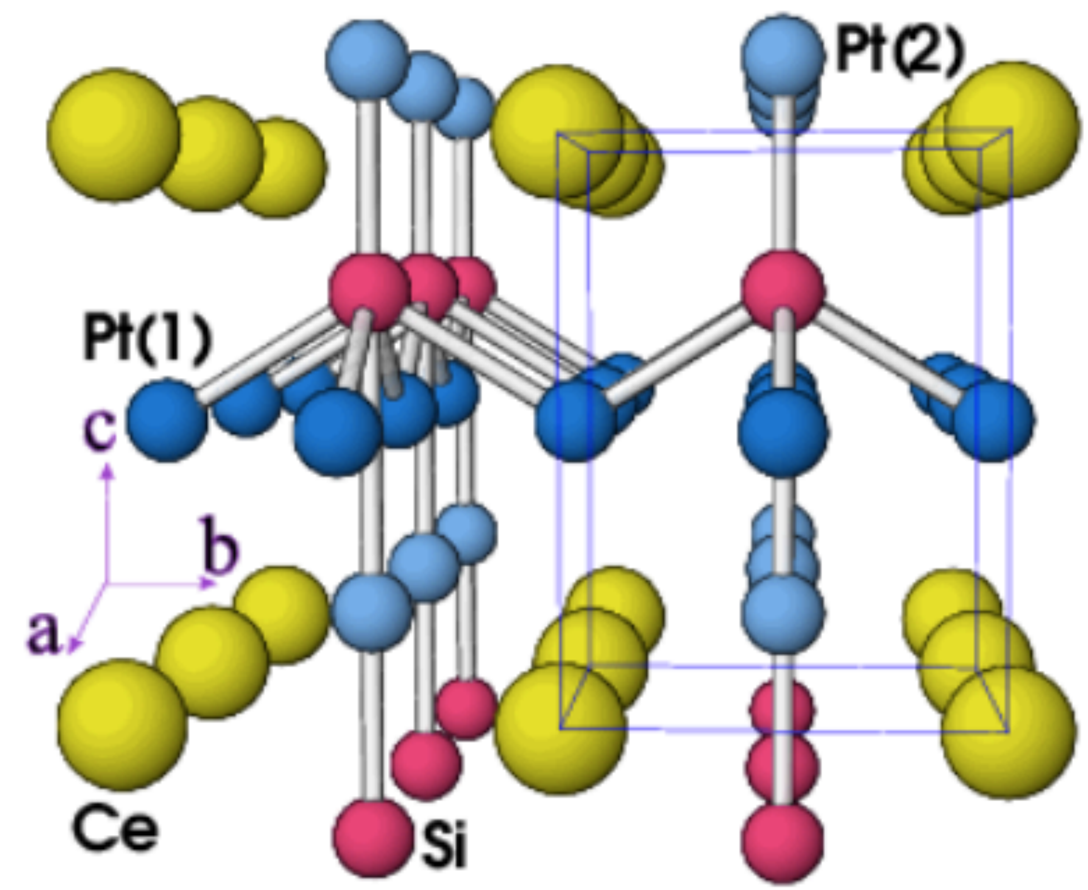
# Plan

- Non-Centrosymmetric Superconductors (NCS)
- Novel EM response
- Fluctuations

# NCS

- Lack of an inversion center.
- Large class: weakly correlated, strongly correlated, two-dimensional materials, and topological superconductors.
- Unusual pairing phase and non-trivial transport properties.
- Lack of inversion allows for singlet-triplet mixing.




$CePt_3Si$ - P4mm; $CePt_3B$-type

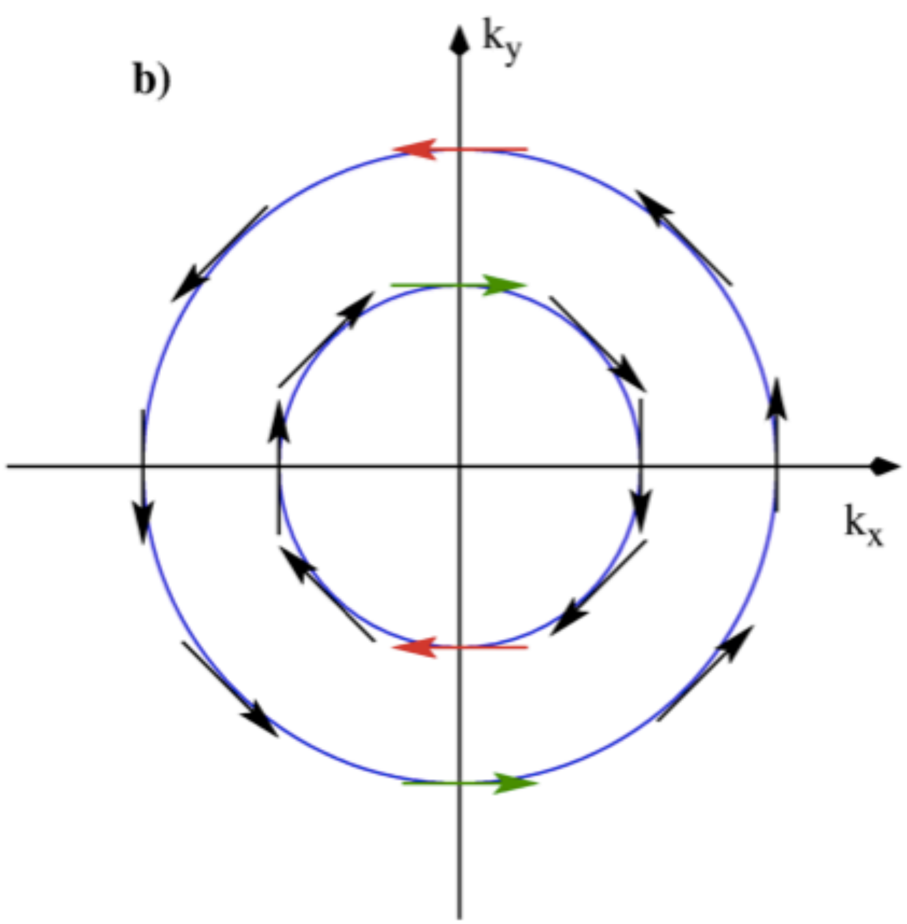
*Source: PhysRevLett.92.027003*

# Spin-Orbit Coupling

- Bulk asymmetry induces a Anti-symmetric Spin Orbit Coupling (ASOC)

$$H(k) = \frac{k^2}{2m} + \vec{\gamma}(\vec{k}) \cdot \vec{\sigma} \qquad \gamma(-\vec{k}) = -\gamma(\vec{k})$$

- Exact nature depends strongly on the symmetry of the crystal.




Examples:

Cubic: $H_{ASOC}: \ \alpha_{xx}(k_x\sigma_x + k_y\sigma_y + k_z\sigma_z)$

$D_3: \quad H_{ASOC}: \alpha_{xx}(k_x\sigma_x + k_y\sigma_y) + \alpha_{zz}k_z\sigma_z$

*Source: arXiv:1609.05953*

# Model

- BCS model with spin orbit coupling term

$$H = \sum_{\vec{x},\sigma} a^{\dagger}_{\sigma}(x)\, H(-i\nabla - e\vec{A})\, a_{\sigma}(x) - V a^{\dagger}_{\uparrow} a^{\dagger}_{\downarrow} a_{\downarrow} a_{\uparrow} + \sum_{\vec{x},\alpha,\beta} a^{\dagger}_{\alpha} \left[ (\vec{\gamma}(-i\nabla - e\vec{A}) - \mu_B \vec{B}) \cdot \vec{\sigma}_{\alpha\beta} \right] a_{\beta}$$

Samoilenka, Babaev

Ref: PRB 102, 184517 (2020)

- $\vec{\gamma}(\vec{k}) = \gamma_0 \vec{k}$ (cubic O, $Li_2Pt_3B$)

- Goal: Focus on EM response, Construct GL

$$Z = \int D[a^{\dagger}, a] e^{-S} \xrightarrow{\text{Mean field}} F[\Delta] = -\frac{1}{\beta} lnZ$$

$$S = \int_0^{\beta} d\tau d\vec{x} \sum_{\alpha,\beta=\downarrow\uparrow} a^{\dagger}_{\alpha} (\mathbf{h} \cdot \boldsymbol{\sigma}_{\alpha\beta}) a_{\beta} - V a^{\dagger}_{\uparrow} a^{\dagger}_{\downarrow} a_{\downarrow} a_{\uparrow}$$

$$\mathbf{h} = (\partial_{\tau} + H - \mu, \vec{h}), \boldsymbol{\sigma}_{\alpha\beta} = (\delta_{\alpha\beta}, \vec{\sigma_{\alpha\beta}}) \; and \quad \vec{h} = \vec{\gamma} - \mu_B \vec{B}$$

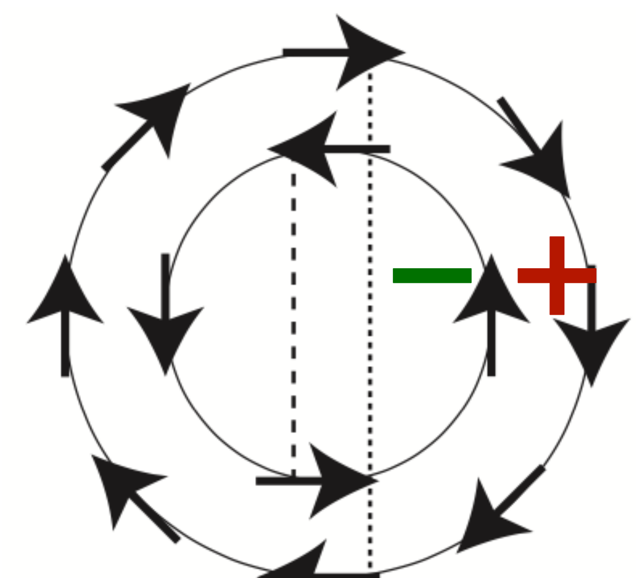
$$F = \int d\vec{r}\, [\alpha\,|\,\Delta\,|^2 + \sum_{a=\pm 1} K_a\,|\,(v_{aF}D^* - 2a\mu_B\vec{B})\cdot\Delta\,|^2 + \beta\,|\,\Delta\,|^4\,] + \frac{1}{2}B^2$$

$$\alpha = N \ln\frac{T}{T_c} \qquad T_c = 2e^{\gamma_{euler}}\cdot\omega_D\frac{e^{-\frac{1}{NV}}}{\pi} \qquad K_a \sim N_a(\epsilon_F) \qquad N = \frac{N_+ + N_-}{2}$$

$$F_{rescaled} = \int d\vec{r}\left[\frac{B^2}{2} + \sum_{a=\pm 1}\frac{\left|\mathscr{D}_a\psi\right|^2}{2\kappa_c} - \left|\psi\right|^2 + \frac{\left|\psi\right|^4}{2}\right]$$

$$\mathscr{D}_a = i\nabla - \vec{A} - (\gamma + a\nu)\vec{B} \qquad \gamma \propto \gamma_0 \qquad \nu \propto \mu_B$$

- Adds $\vec{J}\cdot\vec{B}$ term, generic feature of NCS



Ref: arXiv 1609.05953

$$\sum_a \frac{\mathscr{D}_a^2\psi}{2\kappa_c} - \psi + \psi|\psi|^2 = 0 \qquad \nabla\times[\vec{B} - \sum_a(\gamma + a\nu)\vec{J}_a] = \sum_a \vec{J}_a$$

$$\vec{J}_a = \frac{Re(\psi^*\mathscr{D}_a\psi)}{\kappa_c}$$

# Meissner Effect

- Simplify GL equations: Take London limit ( $|\psi|^2$ const)

$$\nabla \times (\chi^2 \vec{B} + \gamma \vec{j}) + \vec{j} = 0 \quad \vec{j} = \nabla \phi + \vec{A} + \gamma \vec{B}$$

*NCS*

$$\frac{1}{4\pi} \nabla \times (\vec{B} - 4\pi \vec{M}) = \vec{j}$$

*EM*

$$\vec{j} = \nabla \phi + \vec{A}$$

$$\nabla \times \vec{B} + \vec{j} = 0$$

*BCS*

- Current induced magnetisation

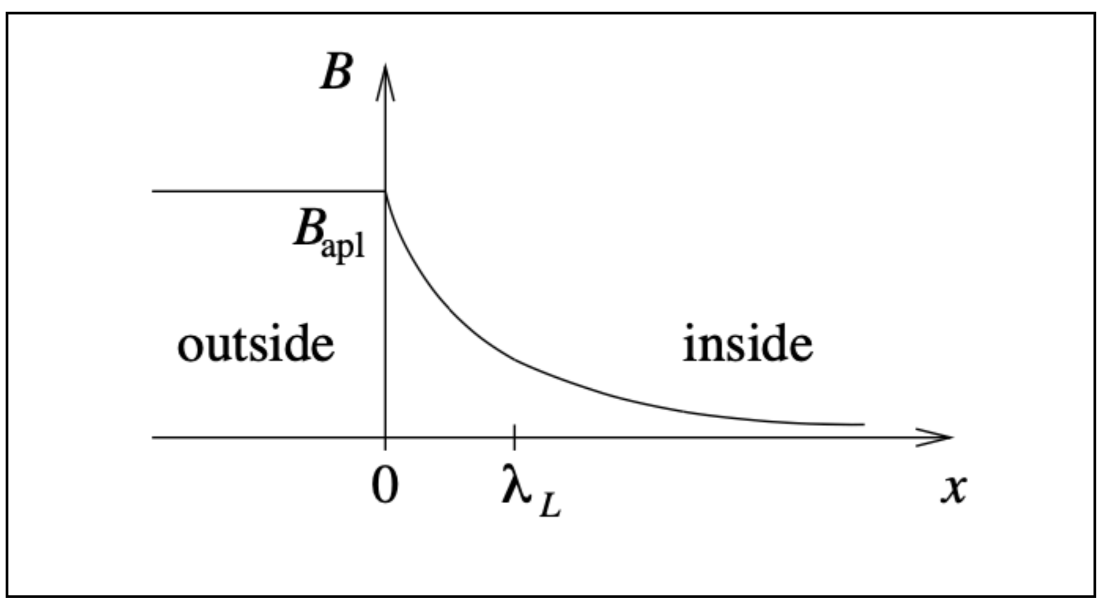
- $\vec{B}$ contributes to current itself $\implies$ new currents now allowed

- Meissner effect modifies: a spiral decay

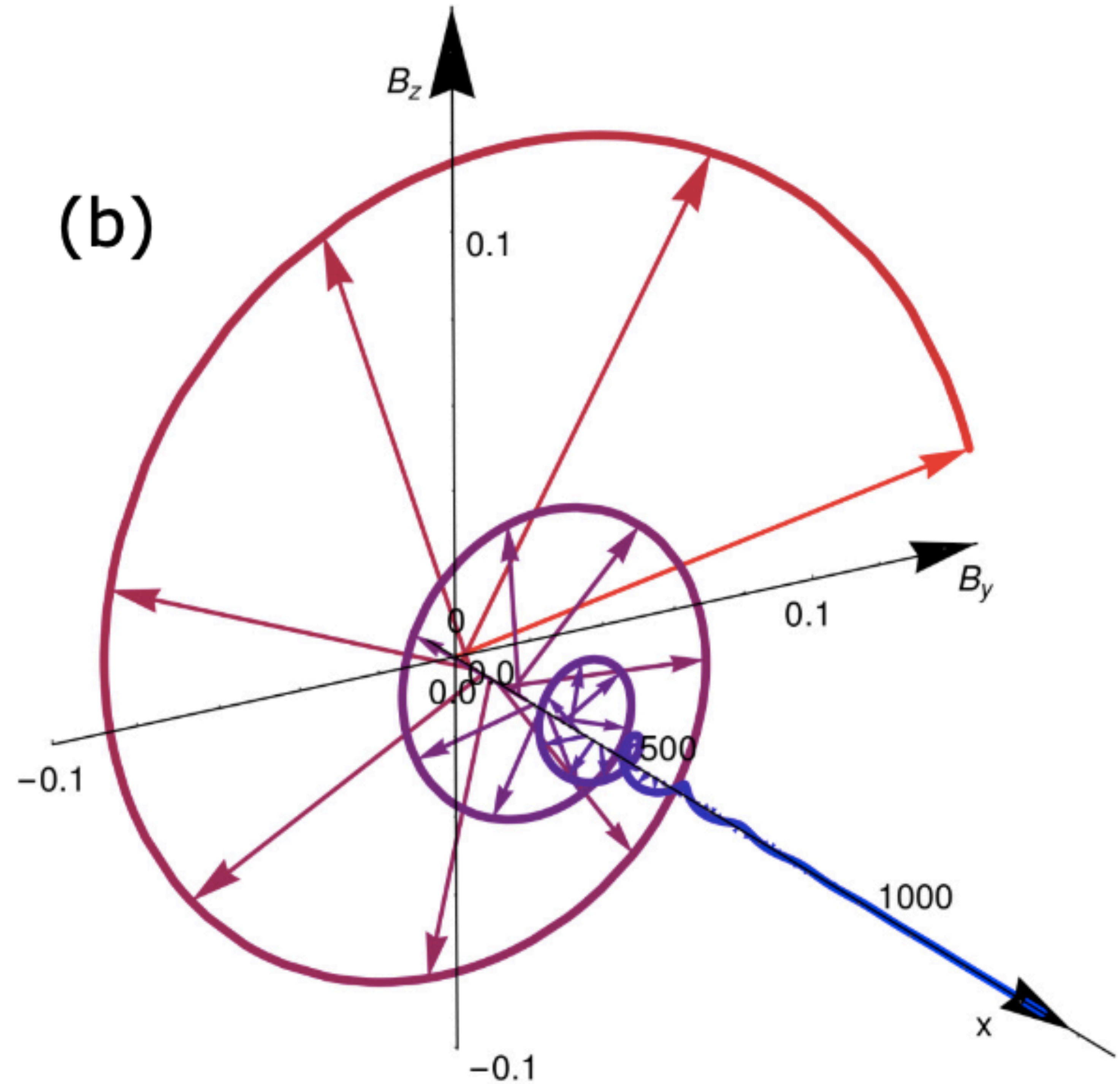
$$\nabla^2 \vec{B} = \frac{1}{\lambda^2} \vec{B} \implies B \sim e^{-x/\lambda} \qquad \tilde{B} = B_z + iB_y = -\frac{i\eta \kappa_c}{2\tilde{\eta}_2} \tilde{H} e^{i\eta x} \propto e^{-\eta_2 x + i\eta_1 x}$$

- $B_z + iB_y \propto e^{-\eta_2 x + i\eta_1 x}$

  where $\eta_1 \propto \gamma \; ( \propto \gamma_0)$ controls <u>handedness</u> and period of rotation of the spiral.

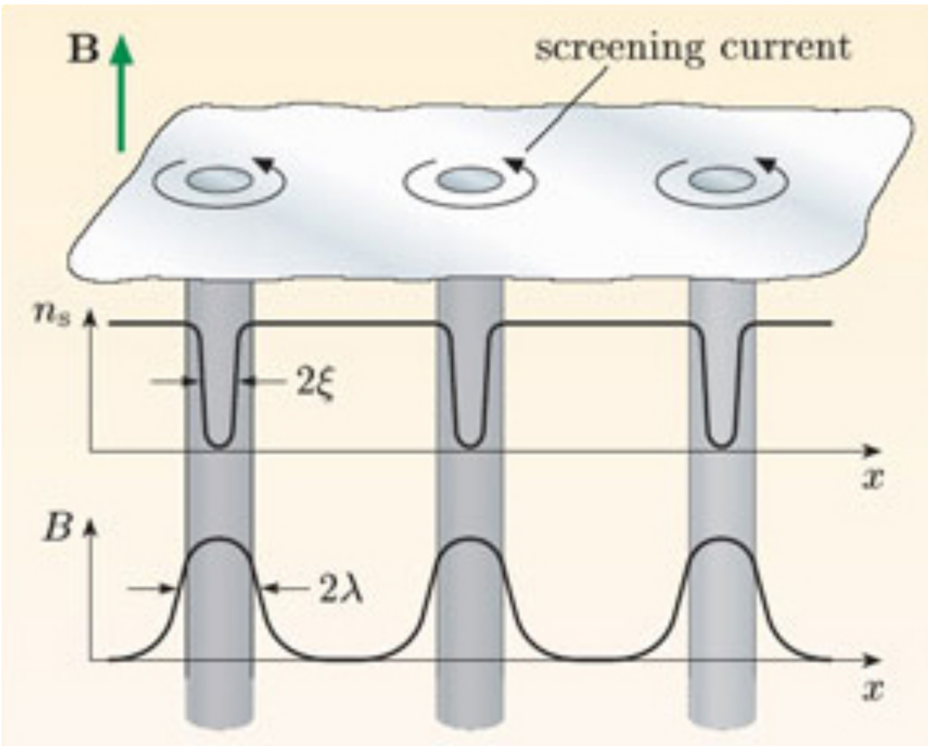



*Usual Meissner Response*



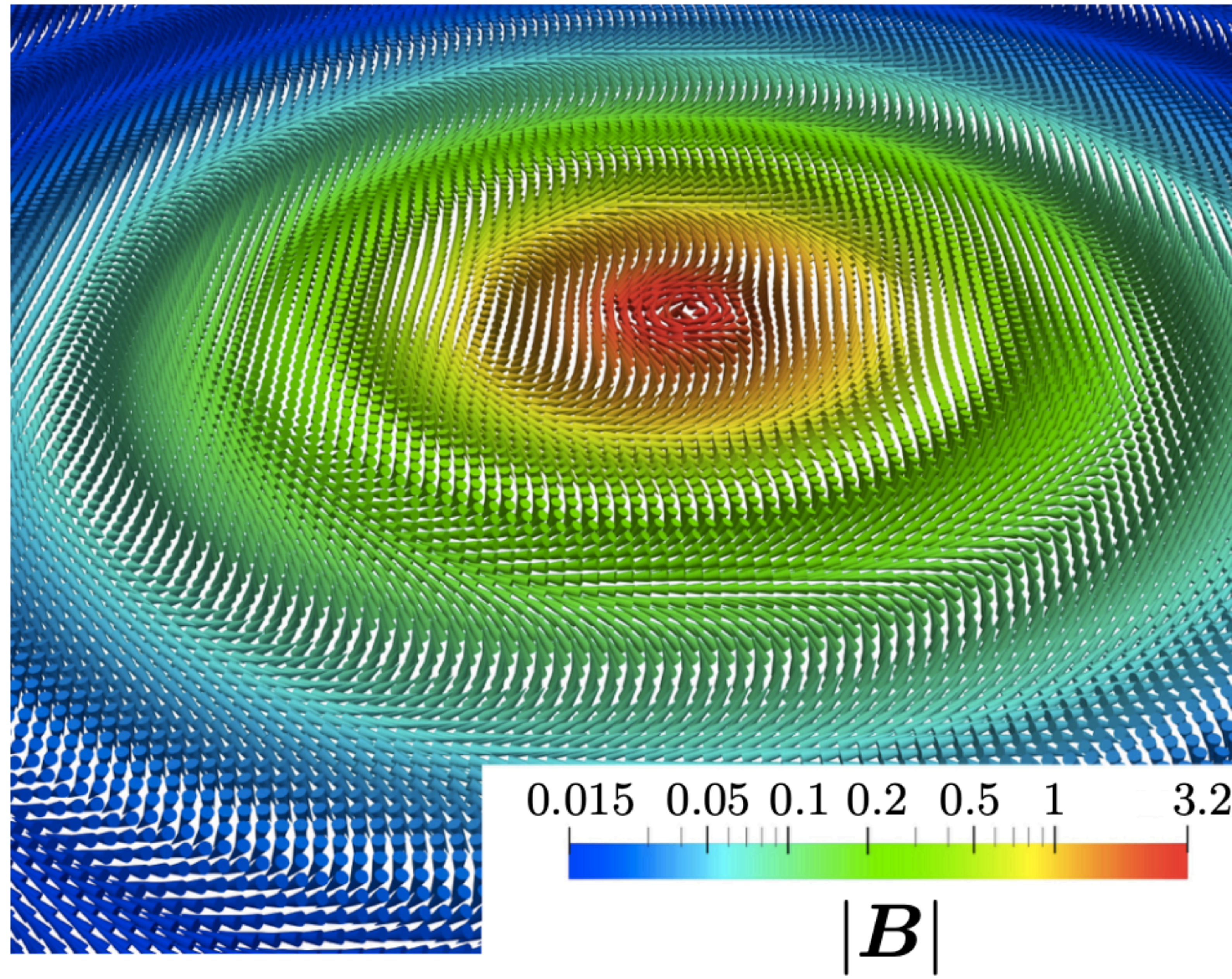

*Meissner effect in NCS*
*Ref: PRB 102, 184517 (2020)*

# Vortex States




*BCS*

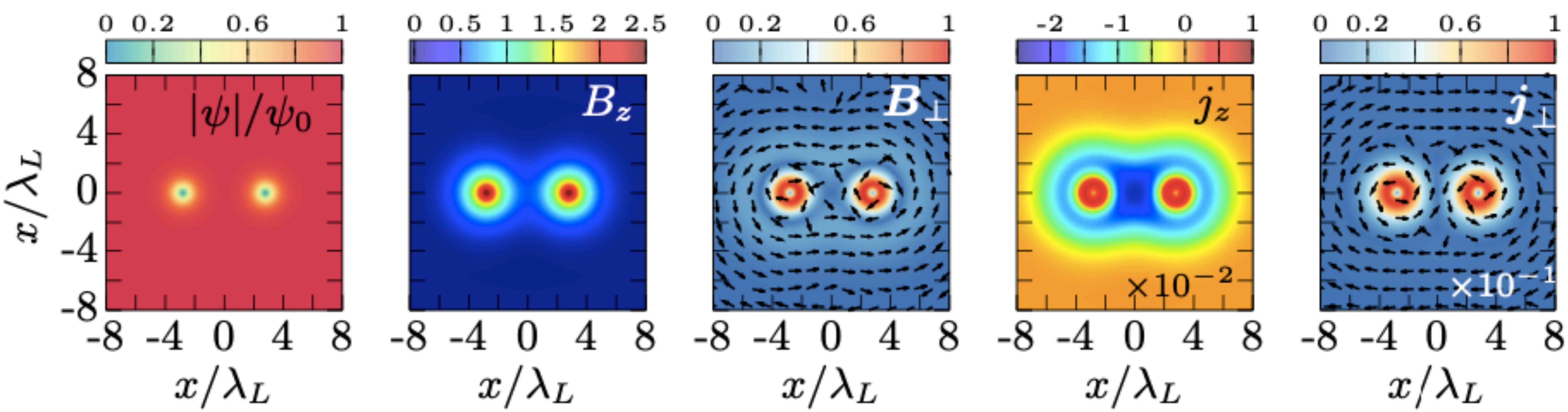
*Fig Ref: The Open University*




*Magnetic field profile around a vortex in NCS.*
*Ref: Phys. Rev. B 102, 184516*

# Inter-vortex interaction

- Inter-vortex interaction is non-monotonic with several minimas $\Longrightarrow$ vortices can form bound states for these distances.
- This can be understood due  to competition between current-current interaction in transverse direction vs longitudinal direction.



# Physical reason

- Can be traced to $\overrightarrow{J} \cdot \overrightarrow{B}$ coupling

$$H(k) = \frac{k^2}{2m} + \vec{\gamma}(\overrightarrow{k}) \cdot \overrightarrow{\sigma} + \mu_B \overrightarrow{B} \cdot \overrightarrow{\sigma}$$

$$\hat{\boldsymbol{\gamma}}(\overrightarrow{k}) = (k_x, k_y, k_z)/|k|$$

$$\Longrightarrow \epsilon_{\pm} \cong k^2/2m \pm \gamma_0 |k| + \hat{\boldsymbol{\gamma}} \cdot \overrightarrow{B}$$

- Apply $B_x \Longrightarrow$ linear term in $k_x$, band centre shifts along $k_x$
- Energetically favourable to form Cooper pairs through the new center of the band as opposed to pairing through the Γ point.
- $|\overrightarrow{k}, + \rangle + |\overrightarrow{-k} + \overrightarrow{q}, + \rangle \longrightarrow \; < a_k a_{-k+q} > \neq 0 \longrightarrow \Delta e^{i2\overrightarrow{q}\cdot\vec{r}}$
- spatially inhomogeneous order parameter ~ associated with a current carrying state.

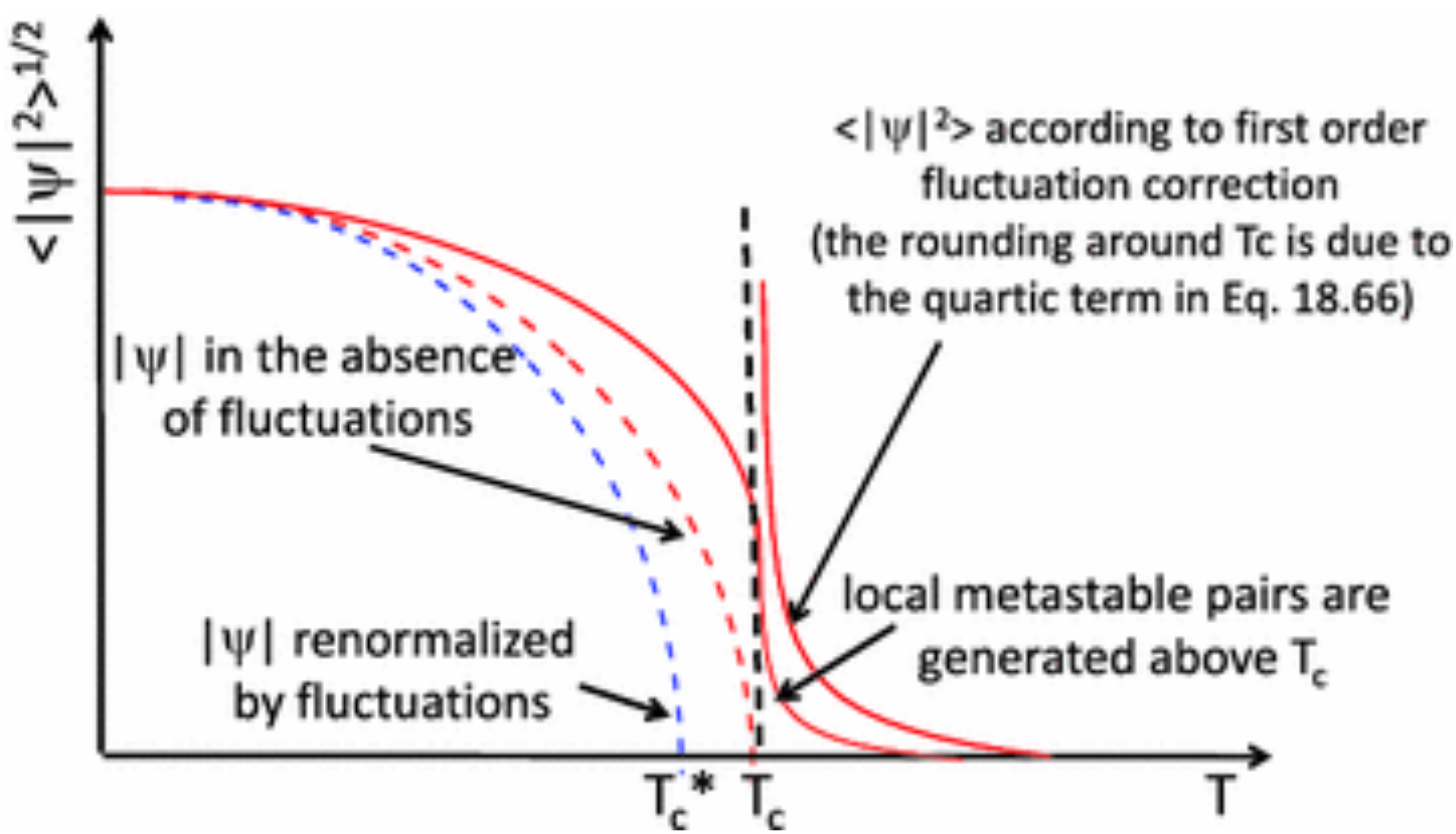
- ASOC, couples $\vec{j}$ with $\overrightarrow{B}$, allows for additional longitudinal current in parallel to $\overrightarrow{B}$.

- Interesting properties: spiral Meissner effect, spiralling vortex structure, and vortex bound states.

- It is then reasonable to ask : What other properties does SOC influence?

# Fluctuations

- GL theory → "mean field description", not universally applicable.
- Superconducting fluctuations above $T_c$ → precursor effects of the SC in normal phase.
- Observables: $\sigma$, $C_V$, $\chi$, etc. may increase considerably in the vicinity of the transition temperature.



# Fluctuational Susceptibility

- To explore ASOC's influence, it's fertile to look at fluctuational contributions to magnetic susceptibility, $\chi_{fluc}$.
- Since fluctuation induced diamagnetism eventually leads to meissner effect as we reduce the temperature, we can expect it to be small wrt diamagnetic susceptibility of a superconductor.
- However, it can be comparable to the value of diamagnetic/ paramagnetic susceptibility of a normal metal.
- For a clean 3d superconductors, $\chi_{fluc}(T \gg T_C) \sim -\chi_P$, Pauli-paramagnetism.

# Calculations

- Take Free energy

$$F = \int d\vec{r}\ [\alpha\,|\,\Delta\,|^2 + \sum_{a=\pm 1} K_a\,|\,(v_{aF}D^* - 2a\mu_B\overrightarrow{B})\cdot\Delta\,|^2 + \beta\,|\,\Delta\,|^4]$$

- Specialise to $T > T_c$ and with weak fluctuations, one gets:

$$F = \frac{Ve^2}{12}B^2\left[\xi + \frac{1}{2}\frac{B^2\gamma^2}{\xi\alpha^2} + \ldots .\right] - \frac{TV}{2\xi\alpha^2}\int_{-\infty}^{\infty}\frac{dk}{2\pi}\int_0^{\infty}\log\frac{\pi k_B T}{A\left(B,k\right)+z}dz$$

$$\chi = -\frac{\partial^2 F}{\partial B^2}$$

# Result

- NCS result:-

$$\chi = \frac{-TVe^2}{6}\left[\xi + \frac{6B^2\gamma^2}{\xi\alpha^{1.5}} + \ldots\right]$$

*At low field*

- BCS result:-

$$\chi^{fluc}_{BCS} = -V \times \frac{1}{6\pi}\frac{e^2}{(hc)^2}T\xi_{GL}$$

*As stated in Physrev.180.527*

- Corrections due to quartic terms, $T_c$ modification etc needs to be taken into account.

- Non-linear response of $\overrightarrow{B}$ has been observed to affect resistivity in NCS systems, asymmetric response (Wakatsuki etal, Sci. Adv., 6, 13, (2020))

# Summary

- Inversion breaking can lead to novel EM response in SCs.
- Coupling between $\vec{j}$ and $\vec{B}$ is a consequence of ASOC.
- Fluctuations feature ASOC's effect → can contribute to observables, expt relevant.

Thank you for your patience!

Although F diverges, taking derivatives to calculate observables like specific heat/susceptibility etc. can converge.

$$\delta C_+ = -\frac{1}{VT_c}\left(\frac{\partial^2 F}{\partial \epsilon^2}\right) = \frac{1}{V}\sum_{\mathbf{k}} \frac{1}{\left(\epsilon + \frac{\mathbf{k}^2}{4m\alpha T_c}\right)^2}.$$

Convergent result

$$\delta C_+ = \frac{1}{8\pi}\frac{(4m\alpha T_c)^{1.5}}{\sqrt{\epsilon}}$$

*Source: arXiv: cond-mat/0109177v1*

# Example

$$F_{GL} = \int a|\Psi|^2 + \frac{b}{2}|\Psi|^4 + \frac{1}{4m}|\nabla\Psi|^2$$

Minimizing the free energy functional we have

$$|\widetilde{\Psi}|^2 = \begin{cases} -\alpha T_c \epsilon / b, & \epsilon < 0 \\ 0, & \epsilon > 0 \end{cases}$$

$$F = (\mathcal{F}[\Psi])_{\min} = \mathcal{F}[\widetilde{\Psi}] = \begin{cases} F_N - \dfrac{\alpha^2 T_c^2 \epsilon^2}{2b} V, & \epsilon < 0 \\ F_N, & \epsilon > 0 \end{cases}$$

*Source: arXiv: cond-mat/0109177v1*

$$\Psi = \varphi_{mF}\left( = 0 \; for \; \epsilon > 0\right) + \psi$$

Decompose the net field into mean field contribution (can be spatially non-uniform)

And thermal fluctuations.

$$F[\Psi] \equiv F[\psi] = \int a|\psi|^2 + \frac{b}{2}|\psi|^4 + \frac{1}{4m}|\nabla\psi|^2$$

Source: arXiv: cond-mat/0109177v1

$$F[\Psi_{\mathbf{k}}] = F_N + \sum_{\mathbf{k}}\left[a + \frac{\mathbf{k}^2}{4m}\right]|\Psi_{\mathbf{k}}|^2$$

$$Z = \prod_{\mathbf{k}}\int d^2\Psi_{\mathbf{k}} \exp\left\{-\alpha(\epsilon + \frac{\mathbf{k}^2}{4m\alpha T_c})|\Psi_{\mathbf{k}}|^2\right\} \qquad F(\epsilon > 0) = -T\ln Z = -T\sum_{\mathbf{k}}\ln\frac{\pi}{\alpha\left(\epsilon + \frac{\mathbf{k}^2}{4m\alpha T_c}\right)}.$$

$$\delta C_+ = \frac{1}{8\pi}\frac{(4m\alpha T_c)^{1.5}}{\sqrt{\epsilon}}$$

# Review slides

# ASOC form for different symmetries

| Point Group | Representation | $\vert\uparrow\rangle$ | $\boldsymbol{\gamma}(\mathbf{k}) \cdot \vec{\tilde{\sigma}}$ |
|---|---|---|---|
| $C_1$ | $\Gamma_2$ | $\vert 1/2, 1/2\rangle$ | $\sum_{i,j=x,y,z} a_{i,j} k_i \sigma_j$ |
| $C_2$ | $\Gamma_3 \oplus \Gamma_4$ | $\vert 1/2, 1/2\rangle$ | $\alpha_{zz} k_z \sigma_z + \alpha_{xx} k_x \sigma_x + \alpha_{yy} k_y \sigma_y + \alpha_{xy} k_x \sigma_y + \alpha_{yx} k_y \sigma_x$ |
| $C_s$ | $\Gamma_3 \oplus \Gamma_4$ | $\vert 1/2, 1/2\rangle$ | $\alpha_{xz} k_x \sigma_z + \alpha_{yz} k_y \sigma_z + \alpha_{zx} k_z \sigma_x + \alpha_{zy} k_z \sigma_y$ |
| $D_2$ | $\Gamma_5$ | $\vert 1/2, 1/2\rangle$ | $\alpha_{xx} k_x \sigma_x + \alpha_{yy} k_y S_y + \alpha_{zz} k_z \sigma_z$ |
| $C_{2v}$ | $\Gamma_5$ | $\vert 1/2, 1/2\rangle$ | $\alpha_{xy} k_x \sigma_y + \alpha_{yx} k_y \sigma_x + \alpha_3 k_x k_y k_z \sigma_z$ |
| $C_4$ | $\Gamma_5 \oplus \Gamma_6$ | $\vert 1/2, 1/2\rangle$ | $\alpha_{xx}(k_x \sigma_x + k_y \sigma_y) + \alpha_{xy}(k_x \sigma_y - k_y \sigma_x) + \alpha_{zz} k_z \sigma_z$ |
| | $\Gamma_7 \oplus \Gamma_8$ | $\vert 3/2, 3/2\rangle$ | $\alpha_{xx}(k_x \sigma_x + k_y \sigma_y) + \alpha_{xy}(k_x \sigma_y - k_y \sigma_x) + \alpha_{zz} k_z \sigma_z$ |
| $S_4$ | $\Gamma_5 \oplus \Gamma_6$ | $\vert 1/2, 1/2\rangle$ | $\alpha_{xx}(k_x \sigma_x + k_y \sigma_y) + \alpha_{xy}(k_x \sigma_y - k_y \sigma_x) + \beta_1 k_z (k_x^2 - k_y^2) \sigma_z + \beta_2 k_z k_x k_y \sigma_z$ |
| | $\Gamma_7 \oplus \Gamma_8$ | $\vert 3/2, 3/2\rangle$ | $\alpha_{xx}(k_x \sigma_x + k_y \sigma_y) + \alpha_{xy}(k_x \sigma_y - k_y \sigma_x) + \beta_1 k_z (k_x^2 - k_y^2) \sigma_z + \beta_2 k_z k_x k_y \sigma_z$ |
| $D_4$ | $\Gamma_6$ | $\vert 1/2, 1/2\rangle$ | $\alpha_{xx}(k_x \sigma_x + k_y \sigma_y) + \alpha_{zz} k_z \sigma_z$ |
| | $\Gamma_7$ | $(x^2 - y^2)\vert 1/2, 1/2\rangle$ | $\alpha_{xx}(k_x \sigma_x + k_y \sigma_y) + \alpha_{zz} k_z \sigma_z$ |
| $C_{4v}$ | $\Gamma_6$ | $\vert 1/2, 1/2\rangle$ | $\alpha_{xy}(k_x \sigma_y - k_y \sigma_x) + \beta k_z k_x k_y (k_x^2 - k_y^2) \sigma_z$ |
| | $\Gamma_7$ | $(x^2 - y^2)\vert 1/2, 1/2\rangle$ | $\alpha_{xy}(k_x \sigma_y - k_y \sigma_x) + \beta k_z k_x k_y (k_x^2 - k_y^2) \sigma_z$ |
| $D_{2d}$ | $\Gamma_6$ | $\vert 1/2, 1/2\rangle$ | $\alpha_{xx}(k_x \sigma_x - k_y \sigma_y) + \beta k_z (k_x^2 - k_y^2) \sigma_z$ |
| | $\Gamma_7$ | $(x^2 - y^2)\vert 1/2, 1/2\rangle$ | $\alpha_{xx}(k_x \sigma_x - k_y \sigma_y) + \beta k_z (k_x^2 - k_y^2) \sigma_z$ |
| $C_3$ | $\Gamma_4 \oplus \Gamma_5$ | $\vert 1/2, 1/2\rangle$ | $\alpha_{xx}(k_x \sigma_x + k_y \sigma_y) + \alpha_{xy}(k_x \sigma_y - k_y \sigma_x) + \alpha_{zz} k_z \sigma_z$ |
| | $\Gamma_6 \oplus \Gamma_6$ | $\vert 3/2, 3/2\rangle$ | $\alpha_x k_z \tilde{\sigma}_x + \alpha_y k_z \tilde{\sigma}_y + \alpha_{zz} k_z \sigma_z$ |
| $D_3$ | $\Gamma_4$ | $\vert 1/2, 1/2\rangle$ | $\alpha_{xx}(k_x \sigma_x + k_y \sigma_y) + \alpha_{zz} k_z \sigma_z$ |
| | $\Gamma_5 \oplus \Gamma_6$ | $\vert 3/2, 3/2\rangle - i\vert 3/2, -3/2\rangle$ | $\alpha_x k_z \tilde{\sigma}_x + \alpha_y k_z \tilde{\sigma}_y + \alpha_{zz} k_z \sigma_z$ |
| $C_{3v}$ | $\Gamma_4$ | $\vert 1/2, 1/2\rangle$ | $\alpha_{xy}(k_x \sigma_y - k_x \sigma_y) + \beta k_y (3k_x^2 - k_y^2) \sigma_z$ |
| | $\Gamma_5 \oplus \Gamma_6$ | $\vert 3/2, 3/2\rangle - i\vert 3/2, -3/2\rangle$ | $\beta_x k_y (3k_x^2 - k_y^2) \tilde{\sigma}_x + \beta_y k_y (3k_x^2 - k_y^2) \tilde{\sigma}_y + \beta_z k_y (3k_x^2 - k_y^2) \sigma_z$ |
| $C_6$ | $\Gamma_7 \oplus \Gamma_8$ | $\vert 1/2, 1/2\rangle$ | $\alpha_{xx}(k_x \sigma_x + k_y \sigma_y) + \alpha_{xy}(k_x \sigma_y - k_y \sigma_x) + \alpha_{zz} k_z \sigma_z$ |
| | $\Gamma_9 \oplus \Gamma_{10}$ | $\vert 5/2, 5/2\rangle$ | $\alpha_{xx}(k_x \sigma_x + k_y \sigma_y) + \alpha_{xy}(k_x \sigma_y - k_y \sigma_x) + \alpha_{zz} k_z \sigma_z$ |
| | $\Gamma_{11} \oplus \Gamma_{12}$ | $\vert 3/2, 3/2\rangle$ | $\beta_1 k_y (3k_x^2 - k_y^2) \tilde{\sigma}_x + \beta_2 k_x (3k_y^2 - k_x^2) \tilde{\sigma}_x + \beta_3 k_y (3k_x^2 - k_y^2) \tilde{\sigma}_y$ $+ \beta_4 k_x (3k_y^2 - k_x^2) \tilde{\sigma}_y + \alpha_{zz} k_z \sigma_z$ |
| $C_{3h}$ | $\Gamma_7 \oplus \Gamma_8$ | $\vert 1/2, 1/2\rangle$ | $\beta_1 k_z [(k_x^2 - k_y^2) \sigma_x + 2k_x k_y \sigma_y] + \beta_2 k_z [-2k_x k_y \sigma_x + (k_x^2 - k_y^2) \sigma_y]$ $+ \beta_3 k_x (3k_y^2 - k_x^2) \sigma_z + \beta_4 k_y (3k_x^2 - k_y^2) \sigma_z$ |
| | $\Gamma_9 \oplus \Gamma_{10}$ | $\vert 5/2, 5/2\rangle$ | $\beta_1 k_z [(k_x^2 - k_y^2) \sigma_x + 2k_x k_y \sigma_y] + \beta_2 k_z [-2k_x k_y \sigma_x + (k_x^2 - k_y^2) \sigma_y]$ $+ \beta_3 k_x (3k_y^2 - k_x^2) \sigma_z + \beta_4 k_y (3k_x^2 - k_y^2) \sigma_z$ |
| | $\Gamma_{11} \oplus \Gamma_{12}$ | $\vert 3/2, 3/2\rangle$ | $\alpha_x k_z \tilde{\sigma}_x + \alpha_y k_z \tilde{\sigma}_y + \beta_1 k_x (3k_y^2 - k_x^2) \sigma_z + \beta_2 k_y (3k_x^2 - k_y^2) \sigma_z$ |
| $D_6$ | $\Gamma_7$ | $\vert 1/2, 1/2\rangle$ | $\alpha_{xx}(k_x \sigma_x + k_y \sigma_y) + \alpha_{zz} k_z \sigma_z$ |
| | $\Gamma_8$ | $y(y^2 - 3x^2)\vert 1/2, 1/2\rangle$ | $\alpha_{xx}(k_x \sigma_x + k_y \sigma_y) + \alpha_{zz} k_z \sigma_z$ |
| | $\Gamma_9$ | $\vert 3/2, 3/2\rangle$ | $\beta_1 k_x (k_x^2 - 3k_y^2) \tilde{\sigma}_x + \beta_2 k_y (k_y^2 - 3k_x^2) \tilde{\sigma}_y + \alpha_{zz} k_z \sigma_z$ |
| $C_{6v}$ | $\Gamma_7$ | $\vert 1/2, 1/2\rangle$ | $\alpha_{xy}(\sigma_x k_y - \sigma_y k_x) + \beta k_z (3k_x^5 k_y - 10k_x^3 k_y^3 + 3k_x k_y^5) \sigma_z$ |
| | $\Gamma_8$ | $x(x^2 - 3y^2)\vert 1/2, 1/2\rangle$ | $\alpha_{xy}(\sigma_x k_y - \sigma_y k_x) + \beta k_z (3k_x^5 k_y - 10k_x^3 k_y^3 + 3k_x k_y^5) \sigma_z$ |
| | $\Gamma_9$ | $\vert 3/2, 3/2\rangle$ | $\beta_1 k_y (k_y^2 - 3k_x^2) \tilde{\sigma}_x + \beta_2 k_x (k_x^2 - 3k_y^2) \tilde{\sigma}_y + \beta_3 k_z (3k_x^5 k_y - 10k_x^3 k_y^3 + 3k_x k_y^5) \sigma_z$ |
| $D_{3h}$ | $\Gamma_7$ | $\vert 1/2, 1/2\rangle$ | $\beta_1 k_z [(k_x^2 - k_y^2) \sigma_x - 2k_x k_y \sigma_y] + \beta_2 k_x (k_x^2 - 3k_y^2) \sigma_z$ |
| | $\Gamma_8$ | $zx(x^2 - 3y^2)\vert 1/2, 1/2\rangle$ | $\beta_1 k_z [(k_x^2 - k_y^2) \sigma_x - 2k_x k_y \sigma_y] + \beta_2 k_x (k_x^2 - 3k_y^2) \sigma_z$ |
| | $\Gamma_9$ | $\vert 3/2, 3/2\rangle$ | $\alpha k_z \tilde{\sigma}_x + \beta_1 k_z (3k_x^5 k_y - 10k_x^3 k_y^3 + 3k_x k_y^5) \tilde{\sigma}_y + \beta_2 k_x (k_x^2 - 3k_y^2) \sigma_z$ |
| $T$ | $\Gamma_5$ | $\vert 1/2, 1/2\rangle$ | $\alpha_{xx}(k_x \sigma_x + k_y \sigma_y + k_z \sigma_z)$ |
| $O$ | $\Gamma_6$ | $\vert 1/2, 1/2\rangle$ | $\alpha_{xx}(k_x \sigma_x + k_y \sigma_y + k_z \sigma_z)$ |
| | $\Gamma_7$ | $xyz\vert 1/2, 1/2\rangle$ | $\alpha_{xx}(k_x \sigma_x + k_y \sigma_y + k_z \sigma_z)$ |
| $T_d$ | $\Gamma_6$ | $\vert 1/2, 1/2\rangle$ | $\beta[k_x (k_y^2 - k_z^2) \sigma_x + k_y (k_z^2 - k_x^2) \sigma_y + k_z (k_x^2 - k_y^2) \sigma_z]$ |
| | $\Gamma_7$ | $f(x)\vert 1/2, 1/2\rangle$ | $\beta[k_x (k_y^2 - k_z^2) \sigma_x + k_y (k_z^2 - k_x^2) \sigma_y + k_z (k_x^2 - k_y^2) \sigma_z]$ |

# Inequalities

For O (cubic)/T(tetrahedral)

$$\vec{\gamma}(\overrightarrow{k}) = \gamma_0 \overrightarrow{k}$$

Also assume that

$$\mu \gg \omega_D \gg T_c$$

$$\gamma_0 k_F \gg \omega_D \gg \mu_B B$$

# ASOC vs Gap

| Compound | Structure | $T_c$ (K) | $\gamma$ (mJ/mol K$^2$) | $H_{c2}$ (T) | $1/T_1(T)$ | KS | $C(T,H)$ | TRSB | $\lambda(T)$ | $E_{ASOC}$ (meV) | $E_{ASOC}/k_B T_c$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $CePt_3Si$ | $P4mm$ | 0.75 | 390 | $2.7\parallel c$, $3.2\parallel a$ | L | C | L | | L | 200[9] | 3095 |
| $LaPt_3Si$ | | 0.6 | 11 | Type I[10,11] | F | | F1 | N | F1 | 200 | 3868 |
| $CeRhSi_3$ | $I4mm$ | 1.05 | 110 | $\sim 30 \parallel c$, $7 \parallel a$ | | | | | | 10 | 111 |
| $CeIrSi_3$ | | 1.6 | 100 | $\sim 45 \parallel c$, $9.5 \parallel a$ | L | C,R | | | | 4 | 29 |
| $CeCoGe_3$ | | 0.64 | 32 | $> 20 \parallel c$, $3.1 \parallel a$ | | | | | | 9[12,13] | 163 |
| $CeIrGe_3$ | | 1.5 | 80 | $> 10 \parallel c$ | | | | | | | |
| UIr | $P2_1$ | 0.13 | 49 | 0.026 | | | | | | | |
| $Li_2Pd_3B$ | $P4_332$ | 7 | 9 | 2 | F | R | F | | F2 | 30 | 50 |
| $Li_2Pt_3B$ | | 2.7 | 7 | 5 | L | C | F/L | | L2 | 200 | 860 |
| $Mo_2Al_3C$ | | 9 | 17.8 | 15 | P | | | N | F1 | | |
| $Y_2C_3$ | $I\bar{4}3d$ | 18 | 6.3 | 30 | F2 | R | F | | L/F2 | 15 | 10 |
| $La_2C_3$ | | 13 | 10.6 | 19 | | C | F1 | | F2 | 30 | 33 |
| $K_2Cr_3As_3$ | $P\bar{6}m2$ | 6.1 | 70-75 | $23 \parallel$, $37 \perp$ | | | | | L | 60 | 114 |
| $Rb_2Cr_3As_3$ | | 4.8 | 55 | 20 | P | | | | | | |
| $Cs_2Cr_3As_3$ | | 2.2 | 39 | 6.5 | | | | | | | |
| BiPd | $P2_1$ | 3.8 | 4 | 0.8 | F1 | | F1 | | F2 | 50 | 153 |
| $Re_6Zr$ | $I\bar{4}3m$ | 6.75 | 26 | 12.2 | | | | Y | F1 | | |
| $Re_3W$ | | 7.8 | 15.9 | 12.5 | | | F1 | N | F1 | | |
| $Nb_xRe_{1-x}$ | | 3.5-8.8 | 3-4.8 | 6-15 | F | R | F1/2 | | F1 | | |
| $Re_{24}Ti_5$ | | 5.8 | 111.8 | 10.75 | | | F1 | | | | |
| $Mg_{10+x}Ir_{19}B_{16-y}$ | $I\bar{4}3m$ | 2.5-5.7 | 52.6 | 0.8 | F1 | R | F1 | | F1/2 | | |
| $Ba(Pt,Pd)Si_3$ | $I4mm$ | 2.3-2.8 | 4.9-5.7 | 0.05-0.10 | | | F1 | | | | |
| $La(Rh,Pt Pd,Ir)Si_3$ | | 0.7-2.7 | 4.4-6 | Type I/0.053 | | | F1 | N | F1 | 17(Rh) | 93(Rh) |
| $Ca(Pt,Ir)Si_3$ | | 2.3-3.6 | 4.0-5.8 | 0.15-0.27 | | | F1 | N | | | |
| $Sr(Ni,Pd,Pt)Si_3$ | | 1.0-3.0 | 3.9-5.3 | 0.039-0.174 | | | F1 | | | | |
| $Sr(Pd,Pt)Ge_3$ | | 1.0-1.5 | 4.0-5.0 | 0.03-0.05 | | | F1 | | | | |

*Table of some known NCS materials.*

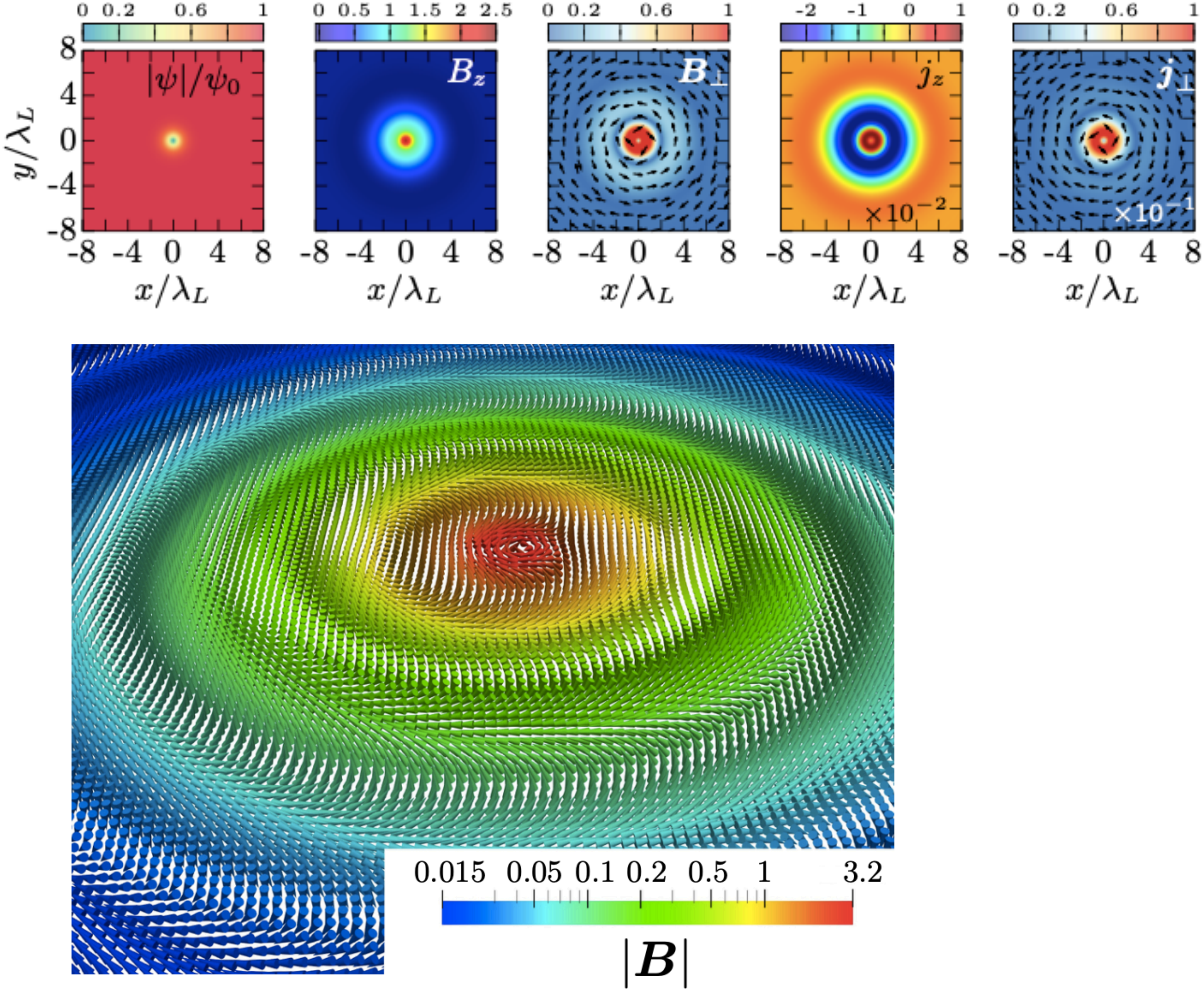
*Source: arXiv: 1609.05953*

# Scalings

$$\vec{x} = \frac{1}{\sqrt{-\alpha}}\left(\frac{\beta}{2e^2}\right)^{\frac{1}{4}}\vec{r}, \quad \Delta = \sqrt{\frac{-\alpha}{2\beta}}\psi$$

$$F = \frac{\sqrt{-\alpha}}{2(2e^2)^{\frac{3}{4}}\beta^{\frac{1}{4}}}F', \quad \vec{A} = \frac{1}{2e}\frac{r}{x}\vec{A}'$$

$$\mathcal{L} = -\eta + \nabla \times \quad \text{with} \quad \eta \equiv \eta_1 + i\eta_2 = \frac{-\gamma + i\chi}{\gamma^2 + \chi^2}.$$

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- SOC, coupling $\vec{j}$ with $\overrightarrow{B}$, allows for additional longitudinal current in parallel to $\overrightarrow{B}$.

- Interesting properties: spiral meissner effect, spiralling vortex structure,  and inter-vortex bound states spawn on account of SOC.

- It is then reasonable to ask : What other properties does SOC affect?

- Recent experiment…. (In the fluctuation….reg)

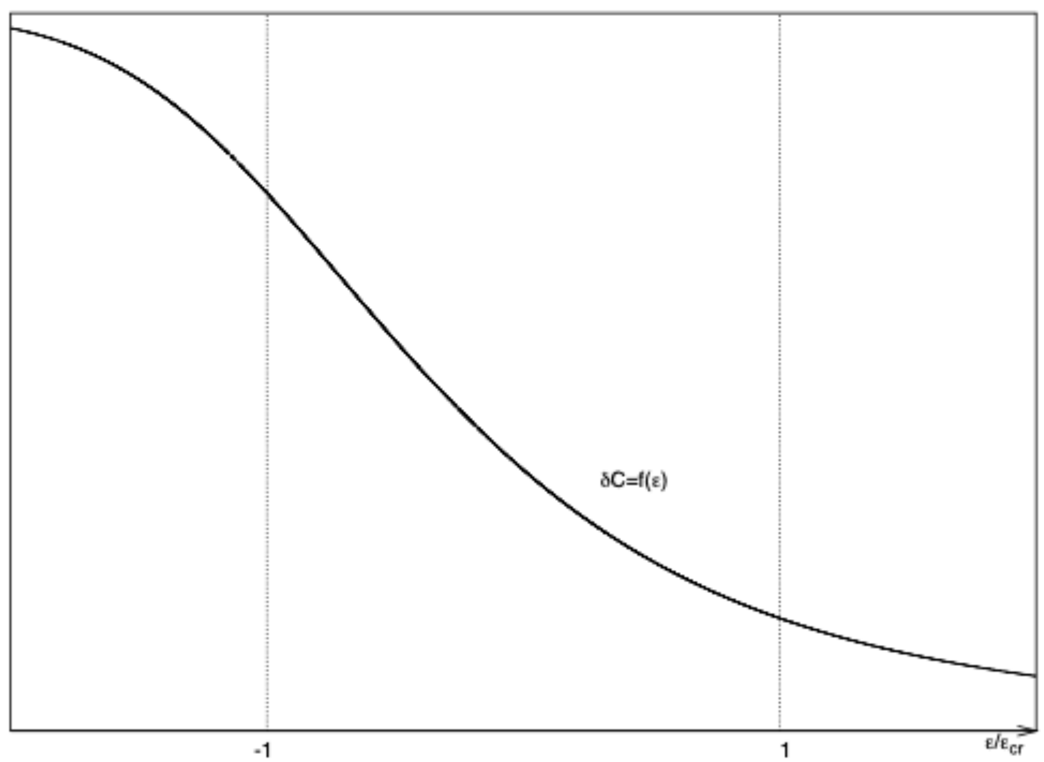



**Fig. 1.** Temperature dependence of the heat capacity of superconducting grains in the region of the critical temperature

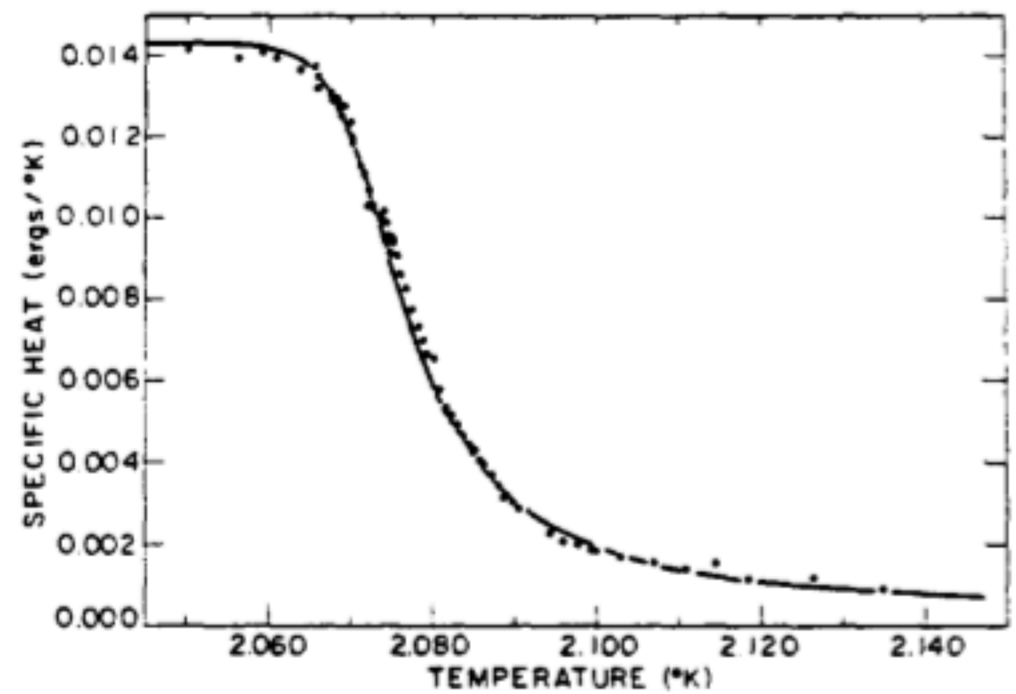
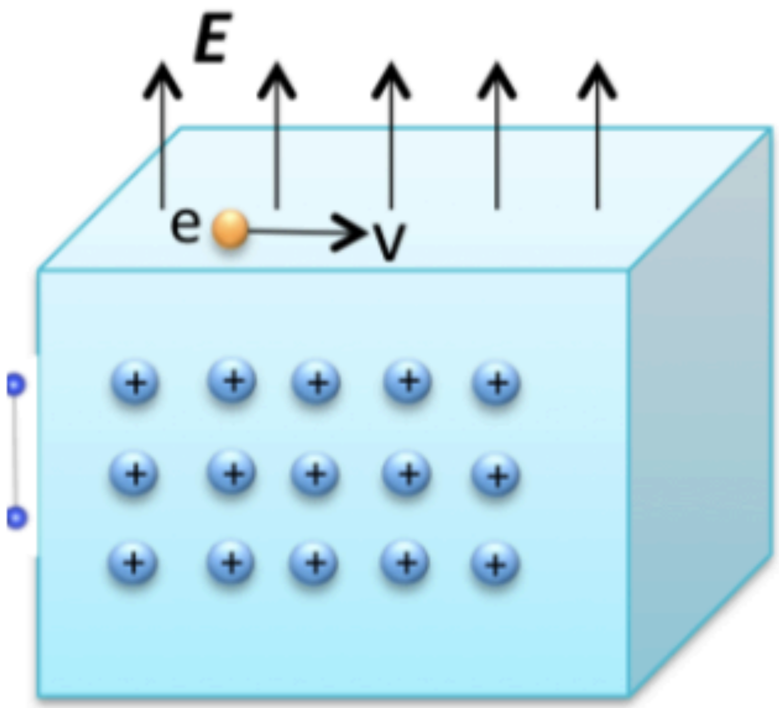



FIG. 1. Comparison of theory with experiment. The dots are the experimental values of the specific heat of a 1350 Å sample of $BiSb_{0.60}$ plotted against temperature. The solid curve is the theoretical curve of specific heat versus temperature determined by the equation $f(C/0.0143) \equiv (0.0143/C-1)+\ln(0.0143/C-1) = 322.6\,T - 669.3$.




$$\hat{H}_R = \frac{k^2}{2m} + \alpha \hat{\mathbf{n}} \cdot (\vec{\sigma} \times \mathbf{k}) = \frac{k^2}{2m} + \alpha\left(\sigma^x k_y - \sigma^y k_x\right)$$

$$t \rightarrow -t: \ \mathbf{k} \rightarrow -\mathbf{k}, \sigma \rightarrow -\sigma$$

$$H_R = \begin{pmatrix} k^2/2m & \alpha\left(k_y + ik_x\right) \\ \alpha\left(k_y - ik_x\right) & k^2/2m \end{pmatrix} \Rightarrow \varepsilon_{\pm} = \frac{k^2}{2m} \pm \alpha k$$

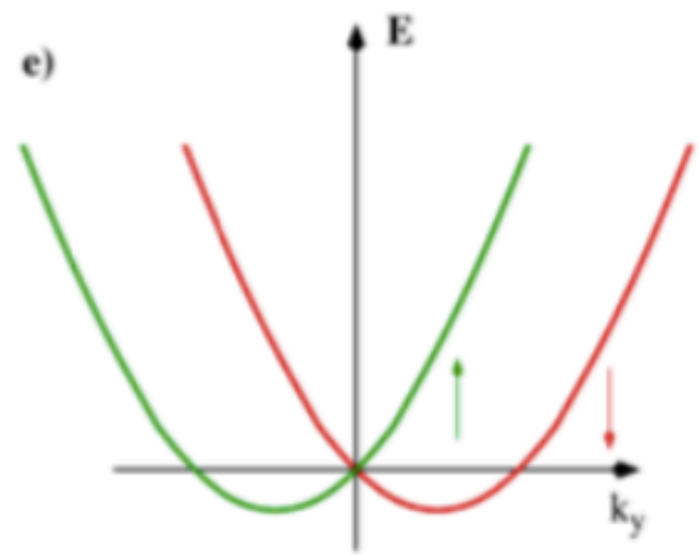
TR - Yes, IR - **Antisymmetric**

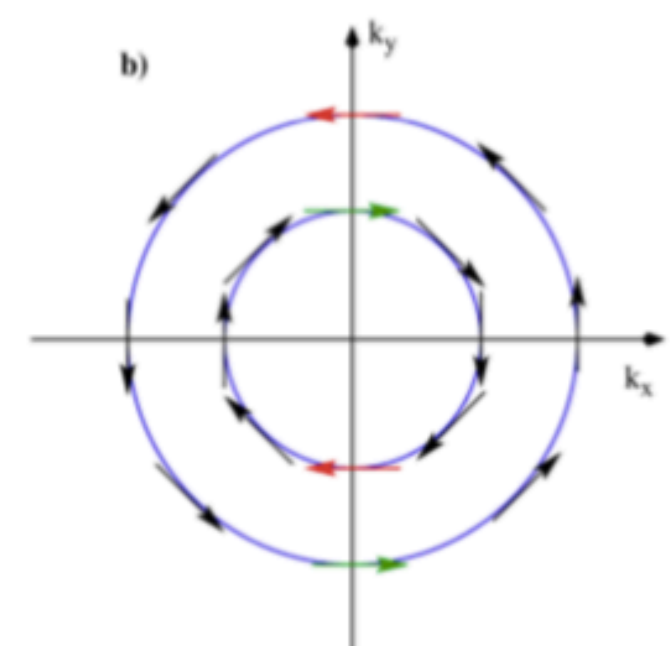
$$H_E = -E_0 z,$$

$$\mathbf{B} = \frac{1}{c^2}\mathbf{E} \times \mathbf{v}.$$

$$H_{SO} = \frac{g\mu_B}{2c^2}(\mathbf{v} \times \mathbf{E}) \cdot \sigma,$$

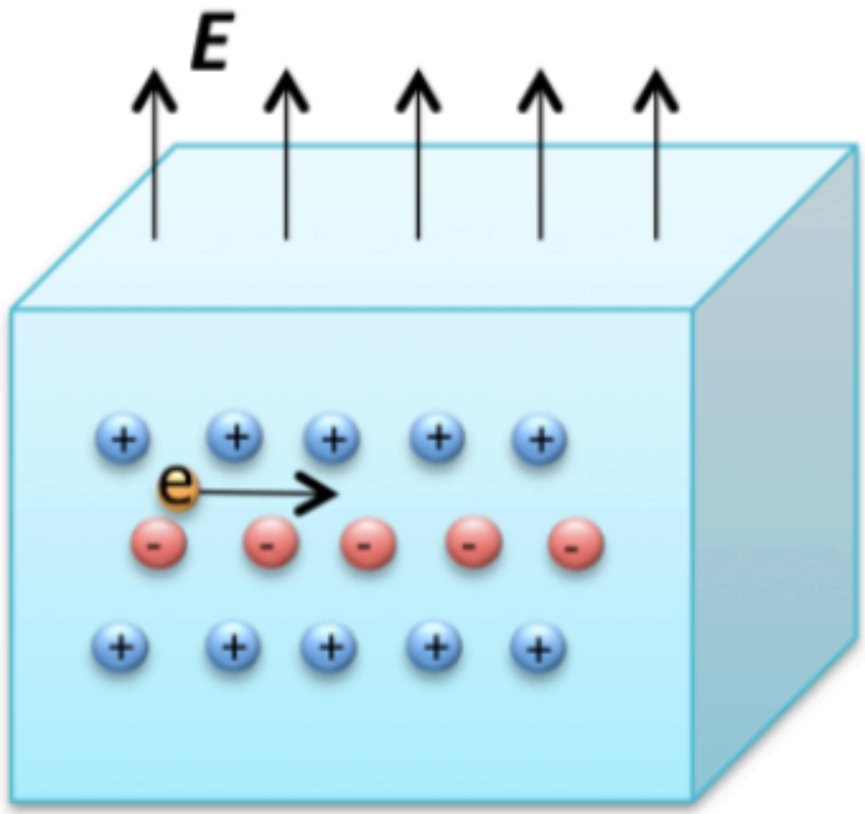
$$H_R = \alpha(\boldsymbol{\sigma} \times \mathbf{p}) \cdot \hat{z},$$




*Source: https://tms16.sciencesconf.org/data/pages/SOC_lecture1.pdf*

$$\hat{H} = \underbrace{\frac{\hat{p}^2}{2m}}_{\text{non-relativistic}} + eV + \underbrace{\frac{\hat{p}^4}{8m^2c^2}}_{\text{K.E. correction}} + \underbrace{\frac{\hbar^2}{8m^2c^2}\nabla^2 V}_{\text{Darwin term}} + \underbrace{\frac{\hbar}{4m^2c^2}\vec{\sigma}\cdot(\vec{\nabla}V \times \hat{\mathbf{p}})}_{\text{SOI}}$$




*Source: https://tms16.sciencesconf.org/data/pages/SOC_lecture1.pdf*

- Bulk asymmetry can also induce a SOC term.
- Exact nature depends strongly on the symmetry of the crystal.

Examples:

Cubic: $H_{ASOC}: \ \alpha_{xx}(k_x\sigma_x + k_y\sigma_y + k_z\sigma_z)$

$D_3: \quad H_{ASOC}: \alpha_{xx}(k_x\sigma_x + k_y\sigma_y) + \alpha_{zz}k_z\sigma_z$

*Source: arXiv:1609.05953*

$$\frac{B^2\gamma^2}{\delta} + \frac{\delta B e}{\pi} \ll a$$

$$a = \alpha \qquad \delta = \xi \cdot \alpha^2$$

$$\kappa_c = \sqrt{\frac{\beta}{2e^2}}\frac{1}{\sum_{a=\pm1} K_a v_{aF}^2}, \quad \vec{H} = \frac{\sqrt{2\beta}}{-\alpha}\vec{\mathcal{H}},$$

$$\gamma = \sqrt{-\alpha}\left(\sum_{a=\pm1} aK_a v_{aF}\right) 2\mu_B \kappa_c \left(\frac{2e^2}{\beta}\right)^{\frac{3}{4}},$$

$$\nu = \sqrt{-\alpha K_+ K_-}\left(\sum_{a=\pm1} v_{aF}\right) 2\mu_B \kappa_c \left(\frac{2e^2}{\beta}\right)^{\frac{3}{4}}.$$

$$\vec{x} = \frac{1}{\sqrt{-\alpha}}\left(\frac{\beta}{2e^2}\right)^{\frac{1}{4}}\vec{r}, \quad \Delta = \sqrt{\frac{-\alpha}{2\beta}}\psi, \quad F = \frac{\sqrt{-\alpha}}{2(2e^2)^{\frac{3}{4}}\beta^{\frac{1}{4}}}F',$$

$$\vec{A} = \frac{1}{2e}\frac{r}{x}\vec{A}'. \tag{28}$$

Thank you for your patience!